

Developing Geo Gebra-Assisted Geometry Media to Support Multimodal Representations of Mathematical Language

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ABSTRACT : Geometry learning requires not only visual and procedural competence but also the ability to comprehend and produce mathematical language through terms, symbols, equations, graphs, and verbal explanations. Presenting these representations separately may hinder meaning-making. This study aimed to develop GeoGebra-assisted geometry media that support multimodal representations of mathematical language for pre-service mathematics teachers. The study employed the 4-D research-and-development model, with reporting focused on the Define, Design, and preliminary Develop stages. Data were obtained from curriculum documents, needs analysis, media prototypes, expert review notes, and revision documentation. The data were analyzed descriptively and qualitatively using lexical, symbolic-syntactic, semantic, discourse, and intermodal dimensions. The findings show that the media integrate geometric terms, coordinates, equations, labels, and graphs within a single dynamic display. Product revisions clarified the relationships between object names and their referents, displayed lengths, midpoints, and equations simultaneously, and facilitated activities involving description, explanation, comparison, and verification. The product has the potential to serve as a language-responsive learning resource because it enables bidirectional movement across verbal, visual, and symbolic representations. However, this study did not test improvements in students' mathematical language or communication; therefore, its effectiveness requires field testing.

KEYWORDS - Mathematical Language, Multimodal Representation, Geogebra, Analytic Geometry, Learning Media.

I. INTRODUCTION

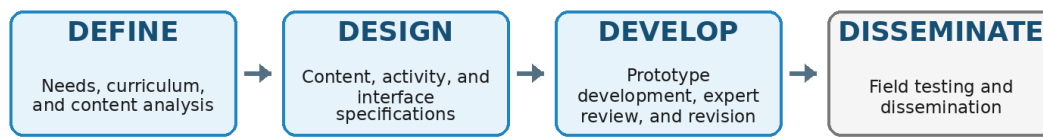
Geometry learning is fundamentally an activity of constructing and communicating meaning. Students do not merely recognize shapes; they also read technical terms, interpret notation, connect equations with objects, describe spatial relationships, and explain why a property holds. Recent scholarship distinguishes language from communication in mathematics education, yet both are understood as resources for meaning-making across modes such as speech, writing, symbols, graphs, diagrams, and gestures (Planas & Pimm, 2024). Therefore, difficulties in learning geometry do not arise solely from the abstract nature of geometric objects, but also from students' inability to connect the linguistic forms through which those objects are represented. Mathematical language comprises lexical, syntactic, semantic, and discourse dimensions. The lexical dimension concerns specialized terms such as point, segment, slope, center, focus, and directrix. The syntactic dimension is evident in the arrangement of symbols and equations; the semantic dimension concerns the meaning of relationships between signs and objects; and the discourse dimension is realized through practices of describing, explaining, comparing, concluding, and justifying. Language-responsive mathematics instruction should involve rich discourse practices, mathematical language routines, connections between linguistic registers and multimodal representations, and instructional sequences that develop language and concepts concurrently (Erath et al., 2021). In analytic geometry, meaning is often distributed across several representations. A point's position is displayed on a coordinate plane, while its identity is expressed through an ordered pair. A line appears as a visual object, but its characteristics are encoded in its slope and equation. Circles, parabolas, ellipses, and hyperbolas likewise require connections among figures, parameters, terms, and algebraic forms. The literature indicates that meaningful geometry experiences should allow students to move among and build connections across visual, symbolic, verbal, contextual, and physical representations (Desai et al., 2021). When these representations are presented separately, students may be able to imitate procedures without explaining their mathematical meaning.

GeoGebra provides a dynamic environment that places algebraic and graphical views within a single workspace. Changes to a point or parameter in the graph are immediately accompanied by symbolic changes. Previous studies have shown that GeoGebra helps learners recall geometric terminology, connect symbolic and visual representations, improve conceptual understanding, and strengthen learning engagement (Thapa et al., 2022; Hidayat et al., 2024; Nalim et al., 2022; Putra et al., 2021; Gurmu et al., 2024). These findings confirm GeoGebra's potential as a multimodal medium rather than merely a drawing tool. Nevertheless, using software does not automatically produce meaningful language learning. A symbol-rich display may remain passive information unless students are asked to name objects, describe changes, explain relationships, or justify conclusions. Recent development research has demonstrated that GeoGebra-assisted learning tools can be directed toward supporting mathematical communication (Ernita & Karyati, 2025). However, studies that specifically examine the organization of lexical, symbolic, visual, and discourse resources within geometry media remain limited. The initial study produced GeoGebra-assisted analytic geometry learning media for pre-service mathematics teachers. In this article, the product is re-examined from a linguistic perspective, particularly through mathematical language and multimodality. This focus does not transform the original data into measures of language proficiency; rather, it examines how the media provide semiotic resources and opportunities for language use. This position is important for keeping the claims aligned with the evidence: the product may be considered potentially supportive of mathematical language, but it cannot be claimed to improve students' language ability before empirical testing is conducted. Based on the foregoing discussion, this study aimed to develop GeoGebra-assisted geometry media that support multimodal representations of mathematical language. The research questions were: (1) what language-related needs should be facilitated in analytic geometry learning; (2) how can these needs be translated into media design; and (3) what linguistic characteristics are evident in the product and its revisions? The novelty of the study lies in positioning GeoGebra media as a multimodal text that connects terms, symbols, equations, labels, and dynamic visualizations.

II. RESEARCH METHODS

This study employed a research-and-development approach using the 4-D model, which comprises Define, Design, Develop, and Disseminate (Thiagarajan et al., 1974). This article reports the Define, Design, and preliminary Develop stages because the product remained a prototype that had undergone an initial revision. The Disseminate stage and effectiveness testing are reserved for subsequent research. The model was used to ensure that design decisions originated from instructional needs and that the product was examined through a staged development process. The study was conducted within the Mathematics Education Study Program at Universitas Muslim Nusantara Al-Washliyah in collaboration with Universiti Teknologi MARA. The research object was GeoGebra-assisted learning media for plane analytic geometry. The content covered coordinate systems, points, segments, lines, midpoints, circles, parabolas, ellipses, and hyperbolas. The product was positioned as multimodal media that present verbal, visual, numerical, and symbolic information in an interconnected manner. Research data were drawn from curriculum documents and analytic geometry materials, needs-analysis notes, GeoGebra prototypes, content and media expert review notes, and documentation of product changes. These data did not include scores of students' language proficiency. Accordingly, the unit of analysis consisted of the linguistic and representational elements embedded in the media design rather than learners' communication performance.

Mathematical language was operationalized through five analytical dimensions. The lexical dimension covered the accuracy and consistency of terminology. The symbolic-syntactic dimension included notation, coordinates, and equation structures. The semantic dimension examined the connection between signs and the objects or relationships they represent. The discourse dimension covered opportunities to describe, explain, compare, conclude, and verify. The multimodal dimension examined movement and connections among verbal, visual, numerical, and symbolic representations. The research instruments consisted of a needs-analysis sheet, a media specification matrix, an expert review sheet, and a revision documentation form developed from these five dimensions. Data were analyzed descriptively and qualitatively through reduction, coding, display, and conclusion drawing. Identified needs were mapped onto design decisions; the initial prototype was compared with the revised product; and each change was interpreted according to its linguistic function. The credibility of the analysis was strengthened through triangulation among content documents, product displays, and expert review notes, as well as through discussions among the researchers. The analysis was not intended to demonstrate improved learning outcomes, but to assess the coherence of the media design as a resource for mathematical language.



Scope of this article: Define, Design, and preliminary Develop stages

Fig 1. Development flow of the mathematical-language-oriented GeoGebra media

III. RESULTS AND DISCUSSION

Results of the Define Stage: Mathematical Language Needs : The Define stage yielded a needs map showing that visualization problems in geometry are simultaneously linguistic problems. When images, terms, and equations are presented separately, students must independently construct the bridges of meaning among them. Therefore, the developed media needed to display term-object correspondence, visual-symbolic relationships, measurement information, and activities that prompted students to produce explanations. A summary of the needs and their design implications is presented in Table 1.

Table 1. Mathematical language needs and media design implications

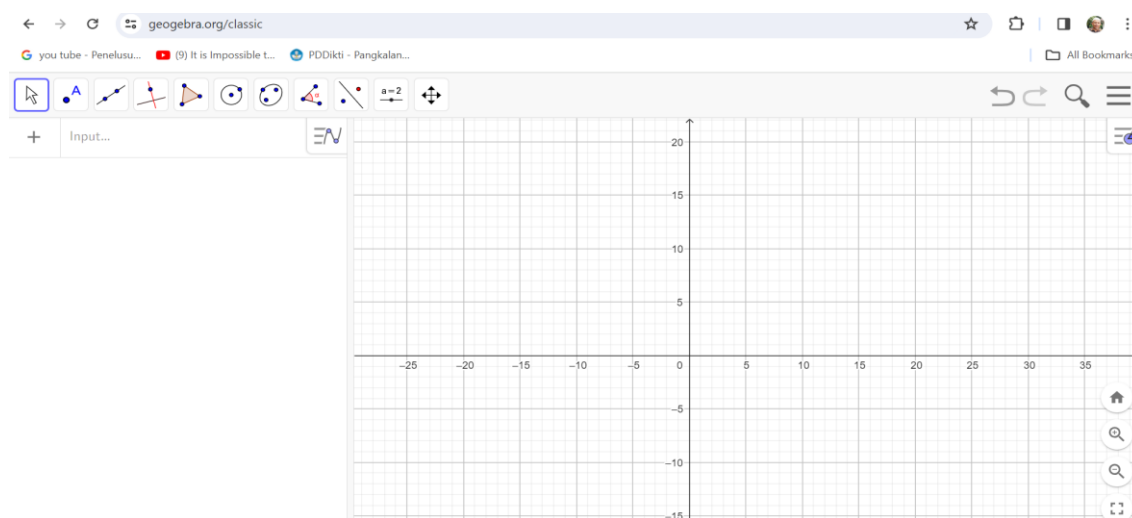
Language-related need	Problem in the initial presentation	Implication for media design
Term-object correspondence	Object names, symbols, and figures were not always displayed together	Labels and terms were attached directly to visual objects
Visual-symbolic connection	Graphs and equations were read as separate information	Algebraic and graphical views were presented simultaneously
Meaning of measurement and position	Lengths, midpoints, coordinates, and slopes were calculated outside the figure	Numerical values were displayed after an object was constructed
Explanatory practice	Activities tended to stop at drawing or calculation	The media included prompts to describe and explain changes
Intermodal translation	Students had limited support for moving from graphs to symbols and verbal language	Activities were designed bidirectionally: observing, writing, explaining, and verifying

These findings broaden the interpretation of difficulties in geometry. The limitations of static images not only make objects difficult to observe, but also restrict meaning-making because symbols and terms do not acquire visual referents that change dynamically. This is consistent with the argument that mathematics instruction should connect technical linguistic registers with multimodal representations rather than merely add lists of terms (Erath et al., 2021). In geometry, the quality of learning is determined by students' ability to translate and explain relationships among representations (Desai et al., 2021).

Results of the Design Stage: Multimodal Media Structure : The Design stage produced specifications for content, activity sequences, and intermodal relationships. The material was organized from coordinate systems to points and segments, lines, circles, and conic sections. Each component was designed not only to generate a figure, but also to display terms, numerical values, equations, and explanatory prompts. Thus, the media were treated as multimodal texts that combine different forms of language to construct a unified mathematical meaning.

Table 2. Specification of representation and mathematical language activities

Component	Multimodal representation	Mathematical language activity
Coordinate system	Terms for axes, quadrants, and origin; scale and Cartesian plane	Naming positions and describing the relationship between coordinates and quadrants
Points and segments	Point labels, ordered pairs, positions, lengths, and midpoints	Translating ordered pairs into positions and explaining changes in length
Lines	Two points, line graph, slope, direction, and equation	Describing direction and explaining the relationship between points and equations
Circle	Center, radius, point on the curve, graph, and equation	Explaining the effects of changing the center or radius
Parabola, ellipse, and hyperbola	Focus, directrix, defining points, curves, and parameters	Comparing terms, properties, and changes in conic-section shapes
Learning activities	Prediction, construction, manipulation, recording, and verification	Using language to describe, explain, compare, and conclude

**Fig 2.** GeoGebra classic interface as a multimodal representational environment

The GeoGebra Classic interface was selected because it displays the toolbar, coordinate plane, algebra view, and input bar on one screen. This arrangement allows a single object to be read through several forms of language. A point can be recognized through its name, ordered pair, and position; a line can be interpreted through its two defining points, graphical form, and equation. This linking provides semantic scaffolding because students encounter not only signs, but also their referents and the changes in meaning that occur when objects are manipulated. This finding is consistent with studies that position GeoGebra as a means of connecting symbolic and visual representations (Thapa et al., 2022; Nalim et al., 2022).

Results of the Develop Stage: Integration and Revision of Representations : During the Develop stage, the design specifications were realized as a prototype using the Point, Line, Segment, Midpoint or Center, Perpendicular Line, Parallel Line, Circle, Ellipse, Parabola, and Hyperbola tools. Each construction produced information that could be read simultaneously. In the points-and-segments topic, students obtained point names, coordinates, positions, and lengths. In the lines topic, the visual object appeared together with its equation. For circles and ellipses, the center or foci, defining points, curves, and equations were presented in dynamic relationships.

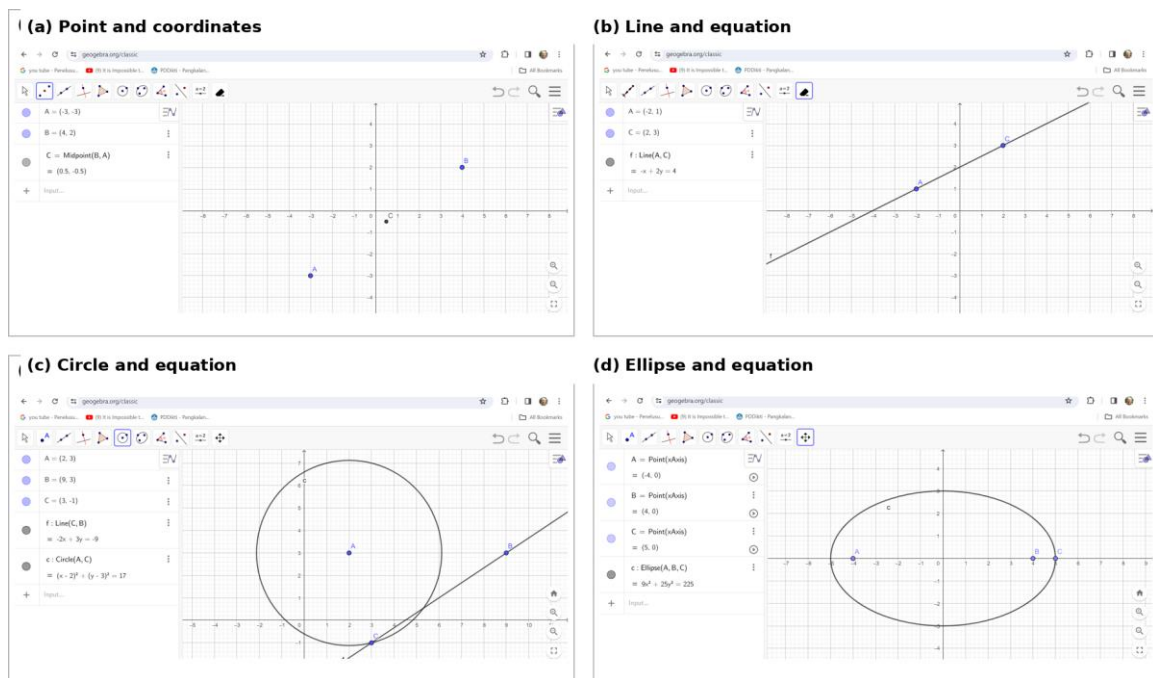


Fig 3. Integration of verbal, visual, numerical, and symbolic representations in the product

Figure 3 shows that the product functions as a multimodal text. Labels A, B, C, and D serve as lexical markers; ordered pairs and equations function as symbolic codes; and points, lines, circles, and ellipses constitute visual representations. When users drag a point, visual and symbolic changes occur simultaneously. This dynamic connection helps maintain referential consistency: object names, positions, values, and equations continue to refer to the same entities even as their parameters change. Expert review and prototype comparison showed that the most meaningful revisions concerned not only graphical neatness but also the clarity of linguistic relationships. In the initial design, construction results had to be explained through separate calculations or drawings. After revision, segment lengths, midpoints, and equations appeared together with the corresponding objects. This simultaneous information strengthened visual-symbolic translation and provided material for students to formulate explanatory statements, such as explaining why moving two points changes a line's slope or how shifting the center affects a circle's equation.

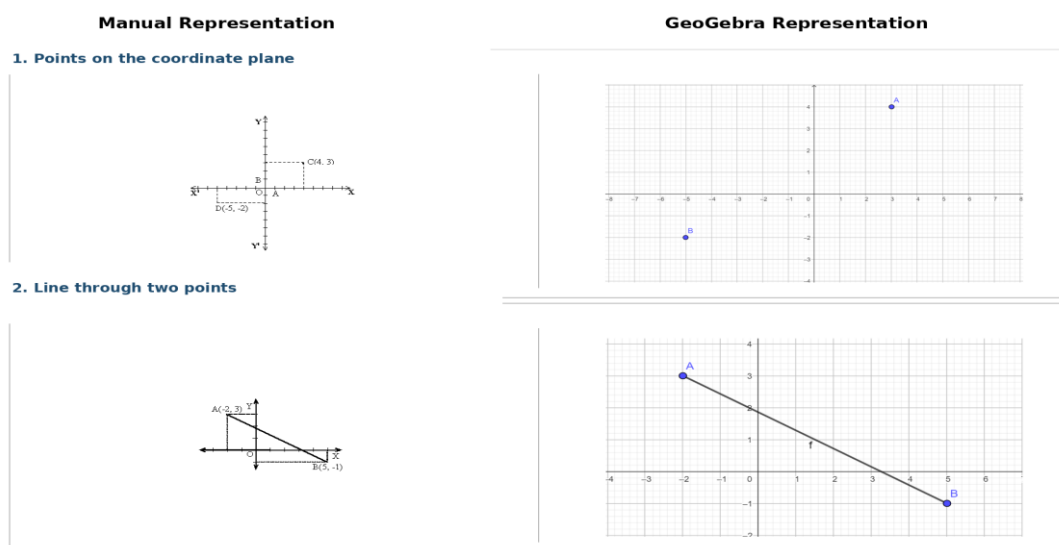


Fig 4. Comparison of manual and revised GeoGebra multimodal representations

Table 3. Product changes and their linguistic significance

Aspect	Before revision	After revision	Linguistic significance
Object name and figure	Figures were drawn manually and labels were weakly connected	Objects were constructed precisely with attached labels	Clarified the relationship between terms and visual referents
Segments and measures	Length was calculated in a separate section	Length appeared together with the segment	Connected measurement terms, numerical values, and objects
Midpoint	Coordinates were calculated and plotted manually	The midpoint was generated using the Midpoint command	Strengthened the visual-symbolic meaning of the term midpoint
Line equation	The equation was calculated after the graph was completed	The equation appeared simultaneously with the line	Supported translation from visual form to symbolic structure
Conic curves	Foci and parameters were difficult to modify	Foci, points, and parameters could be manipulated	Provided material for describing changes and comparing properties

Linguistic Characteristics, Strengths, and Limitations of the Product : The product exhibited five main linguistic characteristics. First, lexical consistency was established through labels and terms attached directly to objects. Second, semantic connectedness emerged because every symbolic change could be traced to a visual change. Third, symbolic-syntactic structures were represented by automatically generated coordinates and equations. Fourth, the media provided a foundation for discourse practices through activities involving description, explanation, comparison, and verification. Fifth, multimodal relationships allowed students to move bidirectionally from graphs to equations, from equations to objects, and from manipulation results to verbal explanations.

Compared with static images, the product's strength lies in its capacity to display semiosis dynamically. In the lines topic, students can move two points, observe changes in the graph and equation, and then explain the relationship between the direction of the line and its coefficients. In the circle topic, changes in the center and radius can be observed together with changes in the algebraic form. In the ellipse and hyperbola topics, changes in the foci can be used to discuss locus definitions. Such activities support the practice of languaging, namely the use of language and semiotic resources to build understanding (Planas & Pimm, 2024). The development results support findings that GeoGebra can help users recall geometric terms and construct symbolic-visual connections (Thapa et al., 2022), while also strengthening conceptual understanding and engagement (Hidayat et al., 2024; Gurmu et al., 2024). However, the specific contribution of this study lies in its analysis of the product's linguistic functions. The media were assessed not only in terms of graphical precision, but also in terms of their capacity to organize technical vocabulary, notation, visual referents, and discourse opportunities within a single learning environment.

This potential can be realized only when lecturers deliberately orchestrate language activities. Automated commands should not replace reasoning or explanation. The media are recommended for use through a prediction-construction-manipulation-description-explanation-verification cycle. Lecturers can ask students to write conjectures, state observed changes, compare two representations, and justify conclusions using precise terminology. This principle is consistent with language-responsive instructional design, which emphasizes rich discourse practices and connections across representations (Erath et al., 2021). This study has several limitations. The product remained at the prototype and initial-revision stage; quantitative validity scores, language-readability data, classroom interaction recordings, and mathematical communication tests were not yet available. Therefore, the study does not conclude that students' language abilities improved. Subsequent testing should involve language or literacy experts, classroom discourse observations, analysis of written responses, and mathematical communication rubrics to determine whether students actually use the identified affordances.

Table 4. Language functions of the media and indicators of student activity

Language function	Example of student activity	Observable indicator
Naming and defining	Pointing to an object and naming its term and defining elements	Uses geometric terms accurately and consistently
Description	Describing changes in position, length, direction, or shape	Expresses visual information in mathematical sentences
Explanation	Explaining why the equation changes when a point is moved	Connects causes, parameter changes, and effects
Comparison	Comparing lines, circles, ellipses, or hyperbolas	Clearly articulates similarities and differences
Verification and argumentation	Matching calculated results with GeoGebra output	Provides reasons and evidence to accept or reject a result
Multimodal translation	Converting a graph into an equation and an equation into an explanation	Connects verbal, visual, numerical, and symbolic representations

IV. CONCLUSION

The study produced a prototype of GeoGebra-assisted geometry media designed to support multimodal representations of mathematical language. The Define stage identified the need for consistent terminology, connections between symbols and objects, and opportunities to produce explanations. The Design stage organized verbal, visual, numerical, and symbolic representations within interconnected activity sequences. The Develop stage produced dynamic media that display labels, coordinates, lengths, midpoints, equations, and graphs simultaneously, while the revisions clarified intermodal translation and discourse functions. The product has the potential to support the naming, description, explanation, comparison, and verification of geometric concepts. Nevertheless, this potential does not constitute evidence of improved mathematical language or communication; its validity, practicality, and effectiveness must be examined through field research using appropriate language-related instruments.

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