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Determinant of P-order and R-ordercubic picture fuzzy soft square matrices

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ABSTRACT: An advanced approach to cubic picture fuzzy soft square matrices (CPFSSMs) are to propose the determinant theory of CPFSSMs. The contribution of this study is to put ahead a new way of expanding the determinant theory of P-order and R-order of CPFSSMs. According to these order operations, we investigated some theoretical properties on the determinant of CPFSSMs.

KEYWORDS: Determinant of a Square Fuzzy Matrix, Cubic Soft Set, Cubic Soft Matrices, Picture Fuzzy Matrix, Cubic Picture-Fuzzy Soft Matrices.

I. INTRODUCTION:

Zadeh[23] introduced the theory of Fuzzy set, consequently he studied the concept of interval-valued Fuzzy sets to capture the uncertainty of membership values. Atanossov's[1] Intuitionistic Fuzzy Sets(IFS) can deal the incomplete information of both the truth membership and non-membership values respectively. Fuzzy matrices were introduced for the first time by Thomason[21], he discussed the convergence of powers of fuzzy matrix. Fuzzy matrices engage in recreation to a vital role in scientific development. In 1999 Molodtsov[15] approaches the theory of Soft Set(SS) which has a rich potential for modelling uncertainty and vagueness. Maji et al[14] introduced the concept of Fuzzy Soft Set(FSS).Bhowmik[2,3] approaches generalized intuitionistic fuzzy matrix and he study some results on intuitionistic fuzzy matrices and intuitionistic circulant fuzzy matrices. In 2003,The adjoint of intuitionistic fuzzy square matrices were introduced by Y.B.Im,Lee and Park[9]. Further Lee[13] approaches the Bipolar valued fuzzy sets and their operations. Overhinnikov[16] proposed to represent the theory of Fuzzy Relation(FR) based on Fuzzy Set Theory.Torra[22] proposed the concept of hesistant fuzzy sets can be used as a functional tool allowing many potential membership degrees of an element to a set, these fuzzy sets having several membership degrees of an element to be possible between zero. Kim[11,12] has constructed the determinant theory for Fuzzy and Boolean matrices,he investigated some properties on the determinant of a Fuzzy matrix. Pal et.al.,[17,18] approachesintuitionistic fuzzy determinant and he also studied about the Interval Valued Fuzzy Matrices(IVFM) with Interval Valued rows and columns. Fermatean fuzzy sets were introduced by Tapansenapati, Ronald and Yager[20]. Fermatean fuzzy matrices were introduced by Silambarasan[19].Picture Fuzzy Set(PFS) was introduced by Cuong[4], which is appropriate for such a situation. Dogra et.al.,[8] proposed to represent the Picture Fuzzy Matrix(PFM), which play an important role for new research in Mathematical Science and Technology. June et.al.,[10] studied the concept of Cubic Sets(CSs) which was very appropriate for such a situation. Cubic Soft Matrix(CSM) was utterly new concept that was coined by Chinnaduraiand Barkavi[5,6], they also extended this concept into Internal and External Cubic Soft Matrix and some theoretical properties are investigated by them. In recent years, Chinnaduraiand Madhanraj [7], defined Cubic Picture Fuzzy Soft Set(CPFSS) and Cubic Picture Fuzzy Soft Matrices(CPFSMs). In this manuscript our intention is to define determinant of cubic picture fuzzy soft square matrices. Furthermore, we scrutinized some desirable properties on determinants of CPFSSM.

II. PRELIMINARIES

Definition 2.1. [5]

Let U ≠ϕ be a non-empty set, then the Cubic Set is defined as

$$C = \{< u, [\tilde{C}_j(u), C_j(u)] \in U >\}. \text{Here, } \tilde{C}_j(u) \text{ is an IVFS in } U \text{ and } C_j(u) \text{ is a FS in } U.$$

Definition 2.2. [5]Let U ≠ϕ be a non-empty set, then the Internal Cubic Sett is defined as $C = \{< u, [\underline{C}_j(u), \overline{C}_j(u)], C_j(u) \in U >\}$. Here,$\underline{C}_j(u) \leq C_j(u) \leq \overline{C}_j(u)$.

Definition 2.3. [5]Let U ≠ϕ be a non-empty set, then the External Cubic Sett is defined as, $C = \{< u, [\underline{C}_j(u), \overline{C}_j(u)], C_j(u) \in U >\}$. Here, $[\underline{C}_j(u), \overline{C}_j(u)] \notin C_j(u)$

Definition 2.1. [6]

Let U ≠ϕ be a non-empty set, E be a 'set of parameters' and A⊆E. A cubic picture-fuzzy soft set over Ṵ is defined as a pair (F, C), and F : C → P^U, (F, C) = {F (e) / e ∈ C},
$where F(e) = \{< (\tilde{C}^t_{F(e)}), C^t_{F(e)} >, < (\tilde{C}^i_{F(e)}), C^i_{F(e)} >, < (\tilde{C}^f_{F(e)}), C^f_{F(e)} >\} and$
$0 \leq (\tilde{C}^t_{F(e)}) + (\tilde{C}^i_{F(e)}) + (\tilde{C}^f_{F(e)}) \leq 1 where\ (\tilde{C}^t_{F(e)}), (\tilde{C}^i_{F(e)}), (\tilde{C}^f_{F(e)})$ are closed sub intervals of [0,1].

3. The determinant of Cubic PictureFuzzy Soft Square Matrices

In this Section, we define the determinant of P-order and R-order Cubic picture-fuzzy soft square matrices and given some suitable example.

Definition 3.1.

The determinant $|D^C_P|_P$ $of\ an\ nxn\ CPFSSM$

$|D^C_P|_P = \left(< \left[D^{\underline{\alpha}}_{P_{ij}}, D^{\overline{\alpha}}_{P_{ij}}\right], D^{\alpha}_{p_{ij}} >, < \left[D^{\underline{\beta}}_{P_{ij}}, D^{\overline{\beta}}_{P_{ij}}\right], D^{\beta}_{p_{ij}} >, < \left[D^{\underline{\gamma}}_{P_{ij}}, D^{\overline{\gamma}}_{P_{ij}}\right], D^{\gamma}_{p_{ij}} >\right)$ is defined as,

$$|D^C_P|_P = [\bigvee_{\sigma \in Sn} \left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\underline{\alpha}}_{n\sigma(n)}\right), \left(D^{\underline{\beta}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\underline{\beta}}_{n\sigma(n)}\right), \left(D^{\underline{\gamma}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\underline{\gamma}}_{n\sigma(n)}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{\left(D^{\overline{\alpha}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\overline{\alpha}}_{n\sigma(n)}\right), \left(D^{\overline{\beta}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\overline{\beta}}_{n\sigma(n)}\right), \left(D^{\overline{\gamma}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\overline{\gamma}}_{n\sigma(n)}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{(D^{\alpha}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\alpha}_{n\sigma(n)}), (D^{\beta}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\beta}_{n\sigma(n)}), (D^{\gamma}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\gamma}_{n\sigma(n)}), \right\}]$$

Where S_n is a symmetric group of all permutations of the indices (1,2,3,….n).

Definition 3.2.

The determinant $|D^C_P|_R$ $of\ an\ nxn\ CPFSSM$ $|D^C_P|_R = \left(< \left[D^{\underline{\alpha}}_{P_{ij}}, D^{\overline{\alpha}}_{P_{ij}}\right], D^{\alpha}_{p_{ij}} >, < \left[D^{\underline{\beta}}_{P_{ij}}, D^{\overline{\beta}}_{P_{ij}}\right], D^{\beta}_{p_{ij}} >, < \left[D^{\underline{\gamma}}_{P_{ij}}, D^{\overline{\gamma}}_{P_{ij}}\right], D^{\gamma}_{p_{ij}} >\right)$ is defined as,

$$|D^C_P|_R = [\bigvee_{\sigma \in Sn} \left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\underline{\alpha}}_{n\sigma(n)}\right), \left(D^{\underline{\beta}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\underline{\beta}}_{n\sigma(n)}\right), \left(D^{\underline{\gamma}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\underline{\gamma}}_{n\sigma(n)}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{(D^{\overline{\alpha}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\overline{\alpha}}_{n\sigma(n)}), (D^{\overline{\beta}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\overline{\beta}}_{n\sigma(n)}), (D^{\overline{\gamma}}_{1_{\sigma(1)}} \wedge \ldots \wedge D^{\overline{\gamma}}_{n\sigma(n)}), \right\},$$
$$\bigwedge_{\sigma \in Sn} \left\{(D^{\alpha}_{1_{\sigma(1)}} \vee \ldots \vee D^{\alpha}_{n\sigma(n)}), (D^{\beta}_{1_{\sigma(1)}} \vee \ldots \vee D^{\beta}_{n\sigma(n)}), (D^{\gamma}_{1_{\sigma(1)}} \vee \ldots \vee D^{\gamma}_{n\sigma(n)}), \right\}]$$

Where S_n is a symmetric group of all permutations of the indices (1,2,3,….n).

Definition 3.3.

Let $D^C_P = \left(< \left[D^{\underline{\alpha}}_{P_{ij}}, D^{\overline{\alpha}}_{P_{ij}}\right], D^{\alpha}_{p_{ij}} >, < \left[D^{\underline{\beta}}_{P_{ij}}, D^{\overline{\beta}}_{P_{ij}}\right], D^{\beta}_{p_{ij}} >, < \left[D^{\underline{\gamma}}_{P_{ij}}, D^{\overline{\gamma}}_{P_{ij}}\right], D^{\gamma}_{p_{ij}} >\right) = [p_{ij}]$
$and\ D^C_Q = \left(< \left[D^{\underline{\mu}}_{Q_{ij}}, D^{\overline{\mu}}_{Q_{ij}}\right], D^{\mu}_{Q_{ij}} >, < \left[D^{\underline{\nu}}_{Q_{ij}}, D^{\overline{\nu}}_{P_{ij}}\right], D^{\nu}_{Q_{ij}} >, < \left[D^{\underline{\varsigma}}_{Q_{ij}}, D^{\overline{\varsigma}}_{Q_{ij}}\right], D^{\varsigma}_{Q_{ij}} >\right) = [q_{ij}]$
be two P – ordered Cubic Picture Fuzzy Soft Matrices of order m x n.
ie,[pij] $\subseteq_P$ $[q_{ij}]. Then\ their\ componetwise\ addition\ and\ componentwise\ multipication\ are\ defined\ as,$

$(i)|D^C_P \oplus_P D^C_Q| = < [max\left\{D^{\underline{\alpha}}_{P_{ij}}, D^{\underline{\beta}}_{P_{ij}}, D^{\underline{\gamma}}_{P_{ij}} >, < D^{\underline{\mu}}_{Q_{ij}}, D^{\underline{\nu}}_{Q_{ij}}, D^{\underline{\varsigma}}_{Q_{ij}} >\right\},$
$max\left\{D^{\overline{\alpha}}_{P_{ij}}, D^{\overline{\beta}}_{P_{ij}}, D^{\overline{\gamma}}_{P_{ij}} >, < D^{\overline{\mu}}_{Q_{ij}}, D^{\overline{\nu}}_{Q_{ij}}, D^{\overline{\varsigma}}_{Q_{ij}} >\right\},$
$max\left\{D^{\alpha}_{P_{ij}}, D^{\beta}_{P_{ij}}, D^{\gamma}_{P_{ij}} >, < D^{\mu}_{Q_{ij}}, D^{\nu}_{P_{ij}}, D^{\varsigma}_{P_{ij}} >\right\}] > for\ all\ i, j.$

$(ii)|D^C_P \odot_P D^C_Q| = < [min\left\{D^{\underline{\alpha}}_{P_{ij}}, D^{\underline{\beta}}_{P_{ij}}, D^{\underline{\gamma}}_{P_{ij}} >, < D^{\underline{\mu}}_{Q_{ij}}, D^{\underline{\nu}}_{Q_{ij}}, D^{\underline{\varsigma}}_{Q_{ij}} >\right\},$
$min\left\{D^{\overline{\alpha}}_{P_{ij}}, D^{\overline{\beta}}_{P_{ij}}, D^{\overline{\gamma}}_{P_{ij}} >, < D^{\overline{\mu}}_{Q_{ij}}, D^{\overline{\nu}}_{Q_{ij}}, D^{\overline{\varsigma}}_{Q_{ij}} >\right\},$
$min\left\{D^{\alpha}_{P_{ij}}, D^{\beta}_{P_{ij}}, D^{\gamma}_{P_{ij}} >, < D^{\mu}_{Q_{ij}}, D^{\nu}_{P_{ij}}, D^{\varsigma}_{P_{ij}} >\right\}] > for\ all\ i, j.$

Since $D^C_P \subseteq_P D^C_Q \leftrightarrow D^{\overline{\alpha}}_{P_{ij}} \leq D^{\overline{\mu}}_{Q_{ij}}, D^{\overline{\beta}}_{P_{ij}} \leq D^{\overline{\nu}}_{P_{ij}},\ D^{\overline{\gamma}}_{P_{ij}} \leq D^{\overline{\varsigma}}_{P_{ij}},\ D^{\underline{\alpha}}_{P_{ij}} \leq D^{\underline{\mu}}_{Q_{ij}}, D^{\underline{\beta}}_{P_{ij}} \leq D^{\underline{\nu}}_{Q_{ij}}, D^{\underline{\gamma}}_{P_{ij}} \leq D^{\underline{\varsigma}}_{Q_{ij}}$, and $D^{\alpha}_{p_{ij}} \leq D^{\mu}_{Q_{ij}}, D^{\beta}_{p_{ij}} \leq D^{\nu}_{Q_{ij}}, D^{\gamma}_{p_{ij}} \leq D^{\varsigma}_{Q_{ij}}$.

Definition 3.4.

Let $D_P^C = \left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right) = [p_{ij}]$

and $D_Q^C = \left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right) = [q_{ij}]$

be two R – ordered Cubic Picture Fuzzy Soft Matrices of order m x n.

ie,[pij] $\subseteq_R [q_{ij}]$. *Then their componetwise addition and componentwise multipication are defined as,*

$(i) |D_P^C \oplus_R D_Q^C| =< [max\left\{D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\underline{\gamma}} >, < D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\underline{\nu}}, D_{Q_{ij}}^{\underline{\varsigma}} >\right\},$

$max\left\{D_{P_{ij}}^{\overline{\alpha}}, D_{P_{ij}}^{\overline{\beta}}, D_{P_{ij}}^{\overline{\gamma}} >, < D_{Q_{ij}}^{\overline{\mu}}, D_{Q_{ij}}^{\overline{\nu}}, D_{Q_{ij}}^{\overline{\varsigma}} >\right\},$

$min\left\{D_{P_{ij}}^{\alpha}, D_{P_{ij}}^{\beta}, D_{P_{ij}}^{\gamma} >, < D_{Q_{ij}}^{\mu}, D_{P_{ij}}^{\nu}, D_{P_{ij}}^{\varsigma} >\right\}] > foralli, j.$

$(i) |D_P^C \odot_R D_Q^C| =< [min\left\{D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\underline{\gamma}} >, < D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\underline{\nu}}, D_{Q_{ij}}^{\underline{\varsigma}} >\right\},$

$min\left\{D_{P_{ij}}^{\overline{\alpha}}, D_{P_{ij}}^{\overline{\beta}}, D_{P_{ij}}^{\overline{\gamma}} >, < D_{Q_{ij}}^{\overline{\mu}}, D_{Q_{ij}}^{\overline{\nu}}, D_{Q_{ij}}^{\overline{\varsigma}} >\right\},$

$max\left\{D_{P_{ij}}^{\alpha}, D_{P_{ij}}^{\beta}, D_{P_{ij}}^{\gamma} >, < D_{Q_{ij}}^{\mu}, D_{P_{ij}}^{\nu}, D_{P_{ij}}^{\varsigma} >\right\}] > foralli, j.$

Since $D_P^C \subseteq_R D_Q^C \leftrightarrow D_{P_{ij}}^{\overline{\alpha}} \leq D_{P_{ij}}^{\overline{\mu}}, D_{P_{ij}}^{\overline{\beta}} \leq D_{P_{ij}}^{\overline{\nu}}, \; D_{P_{ij}}^{\overline{\gamma}} \leq D_{P_{ij}}^{\overline{\varsigma}}, \; D_{P_{ij}}^{\underline{\alpha}} \leq D_{Q_{ij}}^{\underline{\mu}}, D_{P_{ij}}^{\underline{\beta}} \leq D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\underline{\gamma}} \leq D_{Q_{ij}}^{\underline{\varsigma}}$, and $D_{p_{ij}}^{\alpha} \geq D_{Q_{ij}}^{\mu}, D_{p_{ij}}^{\beta} \geq D_{Q_{ij}}^{\nu}, D_{p_{ij}}^{\gamma} \geq D_{Q_{ij}}^{\varsigma}$, for all i ,j

Example 3.5.

$$|D_P^C|_P = \begin{bmatrix} < [0.1,0.2], 0.2, [0.1,0.3], 0.1, [0.2,0.3], 0.3 > & < [0.1,0.3], 0.2, [0.1,0.4], 0.1, [0.1,0.2], 0.2 > & < [0.2,0.3], 0.2[0.1,0.3], 0.2, [0.1,0.4], 0.1 > \\ < [0.2,0.3], 0.2, [0.1,0.4], 0.2, [0.1,0.3], 0.3 > & < [0.1,0.2], 0.1, [0.2,0.3], 0.3, [0.1,0.2], 0.2 > & < [0.1,0.2], 0.2[0.1,0.3], 0.3, [0.2,0.3], 0.2 > \\ < [0.1,0.4], 0.4, [0.1,0.2], 0.1, [0.1,0.4], 0.4 > & < [0.1,0.3], 0.2, [0.1,0.3], 0.1, [0.1,0.3], 0.3 > & < [0.1,0.3], 0.1[0.2,0.4], 0.4, [0.1,0.2], 0.2 > \end{bmatrix}$$

$$|D_P^C|_P = [< [0.1,0.2], 0.2, [0.1,0.3], 0.1, [0.2,0.3], 0.3 >] \wedge$$

$$\begin{bmatrix} < [0.1,0.2], 0.1, [0.2,0.3], 0.3, [0.1,0.2], 0.2 >< [0.1,0.2], 0.2, [0.1,0.3], 0.3, [0.2,0.3], 0.2 > \\ < [0.1,0.3], 0.2, [0.1,0.3], 0.1, [0.1,0.3], 0.3 >< [0.1,0.3], 0.1, [0.2,0.4], 0.4, [0.1,0.2], 0.2 > \end{bmatrix}$$

$$\Rightarrow [< [0.1,0.2], 0.2, [0.1,0.3], 0.1, [0.2,0.3], 0.3 >] \wedge \left\{ \begin{matrix} (< [0.1,0.2], 0.1, [0.2,0.3], 0.3, [0.1,0.2], 0.2 > \wedge < [0.1,0.3], 0.1, [0.2,0.4], 0.4, [0.1,0.2], 0.2 >) \\ \vee (< [0.1,0.3], 0.2, [0.1,0.3], 0.1, [0.1,0.3], 0.3 > \wedge < [0.1,0.2], 0.2[0.1,0.3], 0.3, [0.2,0.3], 0.2 >) \end{matrix} \right\}$$

$$\Rightarrow [< [0.1,0.2], 0.2, [0.1,0.3], 0.1, [0.2,0.3], 0.3 >] \wedge \{(< 0.1,0.2],0.1, [0.2,0.3],0.3, [0.1,0.2],0.2 > \vee < [0.1,0.2],0.2, [0.1,0.3],0.1, [0.1,0.3],0.2 >)\}$$

$$\Rightarrow [< [0.1,0.2], 0.2, [0.1,0.3], 0.1, [0.2,0.3], 0.3 > \wedge < [0.1,0.2], 0.2, [0.1,0.3], 0.1, [0.1,0.2], 0.2 >)$$

$$|D_P^C|_P = [< [0.1,0.2],0.2, [0.1,0.3],0.1, [0.1,0.2],0.2 >]$$

Similarly, we can find $|D_P^C|_R$by definition 3.2.

III. PROPERTY FOR DETERMINANT OF P-ORDER AND R-ORDER CUBIC PICTUREFUZZY SOFT SQUARE MATRICES

In this section, we investigated some properties for the determinant of P-order and R-order cubic picture-fuzzy soft square matrices.

THEOREM 4.1.
Let $D_P^C = \left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right) = [p_{ij}]$
and $D_Q^C = \left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right) = [q_{ij}]$ all $\in CPFSSM_{(mxn)}$,
then $\left|D_P^C \oplus_P D_Q^C\right| = |D_P^C|_P \oplus_P |D_Q^C|_P$
Proof:
L.H.S= $\left|D_P^C \oplus_P D_Q^C\right|$
$\Rightarrow [\left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right) \oplus_P$
$\left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right)$
Since $D_P^C \subseteq_P D_Q^C \leftrightarrow D_{P_{ij}}^{\overline{\alpha}} \le D_{P_{ij}}^{\overline{\mu}}, D_{P_{ij}}^{\overline{\beta}} \le D_{P_{ij}}^{\overline{\nu}}, D_{P_{ij}}^{\overline{\gamma}} \le D_{P_{ij}}^{\overline{\varsigma}}, D_{P_{ij}}^{\underline{\alpha}} \le D_{Q_{ij}}^{\underline{\mu}}, D_{P_{ij}}^{\underline{\beta}} \le D_{Q_{ij}}^{\underline{\nu}},$
$D_{P_{ij}}^{\underline{\gamma}} \le D_{Q_{ij}}^{\underline{\varsigma}}$, and $D_{p_{ij}}^{\alpha} \le D_{Q_{ij}}^{\mu}, D_{p_{ij}}^{\beta} \le D_{Q_{ij}}^{\nu}, D_{p_{ij}}^{\gamma} \le D_{Q_{ij}}^{\varsigma}$, for all i ,j by Definition 3.1.
$\left|D_P^C \oplus_P D_Q^C\right| =< \left[max\left\{D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\underline{\gamma}} >, < D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\underline{\nu}}, D_{Q_{ij}}^{\underline{\varsigma}} >\right\}, max\left\{D_{P_{ij}}^{\overline{\alpha}}, D_{P_{ij}}^{\overline{\beta}}, D_{P_{ij}}^{\overline{\gamma}} > , <\right.\right.$
$\left.\left. D_{Q_{ij}}^{\overline{\mu}}, D_{Q_{ij}}^{\overline{\nu}}, D_{Q_{ij}}^{\overline{\varsigma}} >\right\}, max\left\{D_{P_{ij}}^{\alpha}, D_{P_{ij}}^{\beta}, D_{P_{ij}}^{\gamma} >, < D_{Q_{ij}}^{\mu}, D_{P_{ij}}^{\nu}, D_{P_{ij}}^{\varsigma} >\right\}\right] > for\ all\ i, j.$
$\Rightarrow \left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right)$
It is that, $\left|D_P^C \oplus_P D_Q^C\right| = |D_Q^C|_P$ by Definition 3.3
$|D_P^C|_P = \left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right)$ is defined as,

$$|D_P^C|_P = [\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\alpha}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\beta}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\gamma}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\overline{\alpha}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\beta}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\gamma}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\alpha} \wedge \dots \wedge D_{n\sigma(n)}^{\alpha}\right), \left(D_{1_{\sigma(1)}}^{\beta} \wedge \dots \wedge D_{n\sigma(n)}^{\beta}\right), \left(D_{1_{\sigma(1)}}^{\gamma} \wedge \dots \wedge D_{n\sigma(n)}^{\gamma}\right), \right\}]$$

$|D_Q^C|_P = \left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right)$ is defined as,

$$\left|D_Q^C\right|_P = [\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\mu}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\mu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\nu}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\nu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\varsigma}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\varsigma}}\right), \right\},$$
$$\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\overline{\mu}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\mu}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\nu}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\nu}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\varsigma}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\varsigma}}\right), \right\},$$
$$\bigvee_{\sigma\in Sn} \left\{(D_{1_{\sigma(1)}}^{\mu} \wedge \dots \wedge D_{n\sigma(n)}^{\mu}), (D_{1_{\sigma(1)}}^{\nu} \wedge \dots \wedge D_{n\sigma(n)}^{\nu}), (D_{1_{\sigma(1)}}^{\varsigma} \wedge \dots \wedge D_{n\sigma(n)}^{\varsigma}), \right\}]$$

Then $|D_P^C|_P \oplus_P |D_Q^C|_P = | < \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} > |_P \oplus_P | <$
$\left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} > |_P$

$$\Rightarrow [\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\alpha}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\beta}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\gamma}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\overline{\alpha}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\beta}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\gamma}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\alpha} \wedge \dots \wedge D_{n\sigma(n)}^{\alpha}\right), \left(D_{1_{\sigma(1)}}^{\beta} \wedge \dots \wedge D_{n\sigma(n)}^{\beta}\right), \left(D_{1_{\sigma(1)}}^{\gamma} \wedge \dots \wedge D_{n\sigma(n)}^{\gamma}\right), \right\}]$$
$$\oplus_P [\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\mu}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\mu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\nu}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\nu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\varsigma}} \wedge \dots \wedge D_{n\sigma(n)}^{\underline{\varsigma}}\right), \right\},$$
$$\bigvee_{\sigma\in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\overline{\mu}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\mu}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\nu}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\nu}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\varsigma}} \wedge \dots \wedge D_{n\sigma(n)}^{\overline{\varsigma}}\right), \right\},$$

$$\bigvee_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\varsigma}_{n\sigma(n)}),\right\}]$$

By the Definition 3.1

$$\Rightarrow\{[max\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$

$$\bigvee_{\sigma\in Sn}\left\{(D^{\underline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\mu}}_{n\sigma(n)}),(D^{\underline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\nu}}_{n\sigma(n)}),(D^{\underline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\varsigma}}_{n\sigma(n)}),\right\}]$$

$$[max\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\alpha}}_{n\sigma(n)}\right),\left(D^{\overline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\beta}}_{n\sigma(n)}\right),\left(D^{\overline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\gamma}}_{n\sigma(n)}\right)\right\},$$

$$\bigvee_{\sigma\in Sn}\left\{(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}),(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}),(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}),\right\}]$$

$$[\max\bigvee_{\sigma\in Sn}\left\{\left(D^{\alpha}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\alpha}_{n\sigma(n)}\right),\left(D^{\beta}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\beta}_{n\sigma(n)}\right),\left(D^{\gamma}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\gamma}_{n\sigma(n)}\right)\right\}$$

$$\bigvee_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\varsigma}_{n\sigma(n)}),\right\}]\}$$

$$\Rightarrow\ [\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\mu}}_{n\sigma(n)}\right),\left(D^{\underline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\nu}}_{n\sigma(n)}\right),\left(D^{\underline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\varsigma}}_{n\sigma(n)}\right),\right\},$$

$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}\right),\left(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}\right),\left(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}\right),\right\},$$

$$\bigvee_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\varsigma}_{n\sigma(n)}),\right\}]$$

$=\left|D^{C}_{Q}\right|_{P}$. L.H.S=R.H.S

Hence $\left|D^{C}_{P}\oplus_{P}D^{C}_{Q}\right|=|D^{C}_{P}|_{P}\oplus_{P}|D^{C}_{Q}|_{P}$

THEOREM 4.2. Let $D^{C}_{P}=\left(<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>\right)=[p_{ij}]$ *and* $D^{C}_{Q}=\left(<\left[D^{\underline{\mu}}_{Q_{ij}},D^{\overline{\mu}}_{Q_{ij}}\right],D^{\mu}_{Q_{ij}}>,<\left[D^{\underline{\nu}}_{Q_{ij}},D^{\overline{\nu}}_{P_{ij}}\right],D^{\nu}_{Q_{ij}}>,<\left[D^{\underline{\varsigma}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}\right],D^{\varsigma}_{Q_{ij}}>\right)=[q_{ij}]$all $\in CPFSSM_{(mxn)}$, then $\left|D^{C}_{P}\oplus_{R}D^{C}_{Q}\right|=|D^{C}_{P}|_{R}\oplus_{R}|D^{C}_{Q}|_{R}$

Proof:

L.H.S=$\left|D^{C}_{P}\oplus_{R}D^{C}_{Q}\right|$

$$\Rightarrow[\left(<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>\right)\oplus_{R}$$
$$\left(<\left[D^{\underline{\mu}}_{Q_{ij}},D^{\overline{\mu}}_{Q_{ij}}\right],D^{\mu}_{Q_{ij}}>,<\left[D^{\underline{\nu}}_{Q_{ij}},D^{\overline{\nu}}_{P_{ij}}\right],D^{\nu}_{Q_{ij}}>,<\left[D^{\underline{\varsigma}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}\right],D^{\varsigma}_{Q_{ij}}>\right)$$

Since $D^{C}_{P}\subseteq_{R}D^{C}_{Q}\leftrightarrow D^{\overline{\alpha}}_{P_{ij}}\le D^{\overline{\mu}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\le D^{\overline{\nu}}_{P_{ij}},\ D^{\overline{\gamma}}_{P_{ij}}\le D^{\overline{\varsigma}}_{P_{ij}},\ D^{\underline{\alpha}}_{P_{ij}}\le D^{\underline{\mu}}_{Q_{ij}},D^{\underline{\beta}}_{P_{ij}}\le D^{\underline{\nu}}_{Q_{ij}}$, $D^{\underline{\gamma}}_{P_{ij}}\le D^{\underline{\varsigma}}_{Q_{ij}}$, and $D^{\alpha}_{p_{ij}}\ge D^{\mu}_{Q_{ij}},D^{\beta}_{p_{ij}}\ge D^{\nu}_{Q_{ij}},D^{\gamma}_{p_{ij}}\ge D^{\varsigma}_{Q_{ij}}$, for all i ,j by Definition 3.2.

$\left|D^{C}_{P}\oplus_{R}D^{C}_{Q}\right|=<\left[max\left\{D^{\underline{\alpha}}_{P_{ij}},D^{\underline{\beta}}_{P_{ij}},D^{\underline{\gamma}}_{P_{ij}}>,<D^{\underline{\mu}}_{Q_{ij}},D^{\underline{\nu}}_{Q_{ij}},D^{\underline{\varsigma}}_{Q_{ij}}>\right\},max\left\{D^{\overline{\alpha}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}>\ ,<D^{\overline{\mu}}_{Q_{ij}},D^{\overline{\nu}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}>\right\},min\left\{D^{\alpha}_{P_{ij}},D^{\beta}_{P_{ij}},D^{\gamma}_{P_{ij}}>,<D^{\mu}_{Q_{ij}},D^{\nu}_{P_{ij}},D^{\varsigma}_{P_{ij}}>\right\}\right]>for\ all\ i,j.$

$$\Rightarrow\left(<\left[D^{\underline{\mu}}_{Q_{ij}},D^{\overline{\mu}}_{Q_{ij}}\right],D^{\mu}_{Q_{ij}}>,<\left[D^{\underline{\nu}}_{Q_{ij}},D^{\overline{\nu}}_{P_{ij}}\right],D^{\nu}_{Q_{ij}}>,<\left[D^{\underline{\varsigma}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}\right],D^{\varsigma}_{Q_{ij}}>\right)$$

It is that, $\left|D^{C}_{P}\oplus_{R}D^{C}_{Q}\right|=|D^{C}_{Q}|_{R}$ by Definition 3.4

$|D^{C}_{P}|_{R}=\left(<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>\right)$ is defined as,

$$|D^{C}_{P}|_{R}=[\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$

$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\alpha}}_{n\sigma(n)}\right),\left(D^{\overline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\beta}}_{n\sigma(n)}\right),\left(D^{\overline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\gamma}}_{n\sigma(n)}\right),\right\},$$

$$\bigwedge_{\sigma\in Sn}\left\{(D^{\alpha}_{1_{\sigma(1)}}\vee\ldots\vee D^{\alpha}_{n\sigma(n)}),(D^{\beta}_{1_{\sigma(1)}}\vee\ldots\vee D^{\beta}_{n\sigma(n)}),(D^{\gamma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\gamma}_{n\sigma(n)}),\right\}]$$

$|D^C_Q|_R = \left(<\left[D^{\underline{\mu}}_{Q_{ij}},D^{\overline{\mu}}_{Q_{ij}}\right],D^{\mu}_{Q_{ij}}>,<\left[D^{\underline{\nu}}_{Q_{ij}},D^{\overline{\nu}}_{P_{ij}}\right],D^{\nu}_{Q_{ij}}>,<\left[D^{\underline{\varsigma}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}\right],D^{\varsigma}_{Q_{ij}}>\right)$ is defined as,

$$\left|D^C_Q\right|_P = [\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}\right),\left(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}\right),\left(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}\right),\right\},$$
$$\bigwedge_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\varsigma}_{n\sigma(n)}),\right\}]$$

Then, $|D^C_P|_R \oplus_R |D^C_Q|_R = |<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>|_R \oplus_R |<\left[D^{\underline{\mu}}_{Q_{ij}},D^{\overline{\mu}}_{Q_{ij}}\right],D^{\mu}_{Q_{ij}}>,<\left[D^{\underline{\nu}}_{Q_{ij}},D^{\overline{\nu}}_{P_{ij}}\right],D^{\nu}_{Q_{ij}}>,<\left[D^{\underline{\varsigma}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}\right],D^{\varsigma}_{Q_{ij}}>|_R$

$$\Rightarrow [\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{(D^{\overline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\alpha}}_{n\sigma(n)}),(D^{\overline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\beta}}_{n\sigma(n)}),(D^{\overline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\gamma}}_{n\sigma(n)}),\right\},$$
$$\bigwedge_{\sigma\in Sn}\left\{(D^{\alpha}_{1_{\sigma(1)}}\vee\ldots\vee D^{\alpha}_{n\sigma(n)}),(D^{\beta}_{1_{\sigma(1)}}\vee\ldots\vee D^{\beta}_{n\sigma(n)}),(D^{\gamma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\gamma}_{n\sigma(n)}),\right\}] \oplus_R$$
$$[\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}\right),\left(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}\right),\left(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}\right),\right\},$$
$$\bigwedge_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\varsigma}_{n\sigma(n)}),\right\}]$$

By the Definition 3.2

$$\Rightarrow \{[max\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right)\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{(D^{\underline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\mu}}_{n\sigma(n)}),(D^{\underline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\nu}}_{n\sigma(n)}),(D^{\underline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\varsigma}}_{n\sigma(n)}),\right\}]$$
$$[\max\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\alpha}}_{n\sigma(n)}\right),\left(D^{\overline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\beta}}_{n\sigma(n)}\right),\left(D^{\overline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\gamma}}_{n\sigma(n)}\right)\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}),(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}),(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}),\right\}]$$
$$[\min\bigwedge_{\sigma\in Sn}\left\{\left(D^{\alpha}_{1_{\sigma(1)}}\vee\ldots\vee D^{\alpha}_{n\sigma(n)}\right),\left(D^{\beta}_{1_{\sigma(1)}}\vee\ldots\vee D^{\beta}_{n\sigma(n)}\right),\left(D^{\gamma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\gamma}_{n\sigma(n)}\right),\right\},$$
$$\bigwedge_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\varsigma}_{n\sigma(n)}),\right\}]\}$$
$$\Rightarrow \left|D^C_Q\right|_P = \{[\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}\right),\left(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}\right),\left(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}\right),\right\},$$
$$\bigwedge_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\vee\ldots\vee D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\varsigma}_{n\sigma(n)}),\right\}]\}$$

$=\left|D^C_Q\right|_R$. L.H.S=R.H.S

Hence $\left|D^C_P \oplus_R D^C_Q\right| = |D^C_P|_R \oplus_R |D^C_Q|_R$.

THEOREM 4.3.
Let $D_P^C = \left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right) = [p_{ij}]$
and $D_Q^C = \left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right) = [q_{ij}]$ all $\in CPFSSM_{(mxn)}$,
then $\left|D_P^C \odot_P D_Q^C\right| = |D_P^C|_P \odot_P |D_Q^C|_P$

Proof:
L.H.S=$\left|D_P^C \oplus_P D_Q^C\right|$
$\Rightarrow [\left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right) \odot_P$
$\left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right)$
Since $D_P^C \subseteq_P D_Q^C \leftrightarrow D_{P_{ij}}^{\overline{\alpha}} \le D_{P_{ij}}^{\overline{\mu}}, D_{P_{ij}}^{\overline{\beta}} \le D_{P_{ij}}^{\overline{\nu}}, D_{P_{ij}}^{\overline{\gamma}} \le D_{P_{ij}}^{\overline{\varsigma}}, D_{P_{ij}}^{\underline{\alpha}} \le D_{Q_{ij}}^{\underline{\mu}}, D_{P_{ij}}^{\underline{\beta}} \le D_{Q_{ij}}^{\underline{\nu}}$,
$D_{P_{ij}}^{\underline{\gamma}} \le D_{Q_{ij}}^{\underline{\varsigma}}$, and $D_{p_{ij}}^{\alpha} \le D_{Q_{ij}}^{\mu}, D_{p_{ij}}^{\beta} \le D_{Q_{ij}}^{\nu}, D_{p_{ij}}^{\gamma} \le D_{Q_{ij}}^{\varsigma}$, for all i ,j by Definition 3.1.
$\left|D_P^C \odot_P D_Q^C\right| =< \left[min\left\{D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\underline{\gamma}} >, < D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\underline{\nu}}, D_{Q_{ij}}^{\underline{\varsigma}} >\right\}, min\left\{D_{P_{ij}}^{\overline{\alpha}}, D_{P_{ij}}^{\overline{\beta}}, D_{P_{ij}}^{\overline{\gamma}} >, <\right.\right.$
$\left.\left.D_{Q_{ij}}^{\overline{\mu}}, D_{Q_{ij}}^{\overline{\nu}}, D_{Q_{ij}}^{\overline{\varsigma}} >\right\}, min\left\{D_{P_{ij}}^{\alpha}, D_{P_{ij}}^{\beta}, D_{P_{ij}}^{\gamma} >, < D_{Q_{ij}}^{\mu}, D_{P_{ij}}^{\nu}, D_{P_{ij}}^{\varsigma} >\right\}\right] > for\ all\ i, j.$
$\Rightarrow \left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right)$
It is that, $\left|D_P^C \odot_P D_Q^C\right| = |D_P^C|_P$ by Definition 3.3.
$|D_P^C|_P = \left(< \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} >\right)$ is defined as,

$$|D_P^C|_P = [\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\alpha}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\beta}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\gamma}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\overline{\alpha}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\beta}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\gamma}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\alpha} \wedge \ldots \wedge D_{n\sigma(n)}^{\alpha}\right), \left(D_{1_{\sigma(1)}}^{\beta} \wedge \ldots \wedge D_{n\sigma(n)}^{\beta}\right), \left(D_{1_{\sigma(1)}}^{\gamma} \wedge \ldots \wedge D_{n\sigma(n)}^{\gamma}\right), \right\}]$$

$|D_Q^C|_P = \left(< \left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} >\right)$ is defined as,

$$\left|D_Q^C\right|_P = [\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\mu}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\mu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\nu}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\nu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\varsigma}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\varsigma}}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\overline{\mu}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\mu}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\nu}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\nu}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\varsigma}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\varsigma}}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{(D_{1_{\sigma(1)}}^{\mu} \wedge \ldots \wedge D_{n\sigma(n)}^{\mu}), (D_{1_{\sigma(1)}}^{\nu} \wedge \ldots \wedge D_{n\sigma(n)}^{\nu}), (D_{1_{\sigma(1)}}^{\varsigma} \wedge \ldots \wedge D_{n\sigma(n)}^{\varsigma}), \right\}]$$

Then $|D_P^C|_P \odot_P |D_Q^C|_P = | < \left[D_{P_{ij}}^{\underline{\alpha}}, D_{P_{ij}}^{\overline{\alpha}}\right], D_{p_{ij}}^{\alpha} >, < \left[D_{P_{ij}}^{\underline{\beta}}, D_{P_{ij}}^{\overline{\beta}}\right], D_{p_{ij}}^{\beta} >, < \left[D_{P_{ij}}^{\underline{\gamma}}, D_{P_{ij}}^{\overline{\gamma}}\right], D_{p_{ij}}^{\gamma} > |_P \odot_P | <$
$\left[D_{Q_{ij}}^{\underline{\mu}}, D_{Q_{ij}}^{\overline{\mu}}\right], D_{Q_{ij}}^{\mu} >, < \left[D_{Q_{ij}}^{\underline{\nu}}, D_{P_{ij}}^{\overline{\nu}}\right], D_{Q_{ij}}^{\nu} >, < \left[D_{Q_{ij}}^{\underline{\varsigma}}, D_{Q_{ij}}^{\overline{\varsigma}}\right], D_{Q_{ij}}^{\varsigma} > |_P$
$$\Rightarrow [\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\alpha}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\beta}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\gamma}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\overline{\alpha}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\alpha}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\beta}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\beta}}\right), \left(D_{1_{\sigma(1)}}^{\overline{\gamma}} \wedge \ldots \wedge D_{n\sigma(n)}^{\overline{\gamma}}\right), \right\},$$
$$\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\alpha} \wedge \ldots \wedge D_{n\sigma(n)}^{\alpha}\right), \left(D_{1_{\sigma(1)}}^{\beta} \wedge \ldots \wedge D_{n\sigma(n)}^{\beta}\right), \left(D_{1_{\sigma(1)}}^{\gamma} \wedge \ldots \wedge D_{n\sigma(n)}^{\gamma}\right), \right\}] \odot_P$$
$$[\bigvee_{\sigma \in Sn} \left\{\left(D_{1_{\sigma(1)}}^{\underline{\mu}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\mu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\nu}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\nu}}\right), \left(D_{1_{\sigma(1)}}^{\underline{\varsigma}} \wedge \ldots \wedge D_{n\sigma(n)}^{\underline{\varsigma}}\right), \right\},$$

$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}\right),\left(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}\right),\left(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\varsigma}_{n\sigma(n)}),\right\}]$$

By the Definition 3.1

$$\Rightarrow\{[\min\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{(D^{\underline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\mu}}_{n\sigma(n)}),(D^{\underline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\nu}}_{n\sigma(n)}),(D^{\underline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\varsigma}}_{n\sigma(n)}),\right\}]$$
$$[\min\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\alpha}}_{n\sigma(n)}\right),\left(D^{\overline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\beta}}_{n\sigma(n)}\right),\left(D^{\overline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\gamma}}_{n\sigma(n)}\right)\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{(D^{\overline{\mu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\mu}}_{n\sigma(n)}),(D^{\overline{\nu}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\nu}}_{n\sigma(n)}),(D^{\overline{\varsigma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\varsigma}}_{n\sigma(n)}),\right\}]$$
$$[\min\bigwedge_{\sigma\in Sn}\left\{\left(D^{\alpha}_{1_{\sigma(1)}}\vee\ldots\vee D^{\alpha}_{n\sigma(n)}\right),\left(D^{\beta}_{1_{\sigma(1)}}\vee\ldots\vee D^{\beta}_{n\sigma(n)}\right),\left(D^{\gamma}_{1_{\sigma(1)}}\vee\ldots\vee D^{\gamma}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{(D^{\mu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\mu}_{n\sigma(n)}),(D^{\nu}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\nu}_{n\sigma(n)}),(D^{\varsigma}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\varsigma}_{n\sigma(n)}),\right\}]\}$$

$$\Rightarrow|D^{C}_{P}|_{P}=[\bigvee_{\sigma\in Sn}\left\{\left(D^{\underline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\alpha}}_{n\sigma(n)}\right),\left(D^{\underline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\beta}}_{n\sigma(n)}\right),\left(D^{\underline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\underline{\gamma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\overline{\alpha}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\alpha}}_{n\sigma(n)}\right),\left(D^{\overline{\beta}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\beta}}_{n\sigma(n)}\right),\left(D^{\overline{\gamma}}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\overline{\gamma}}_{n\sigma(n)}\right),\right\},$$
$$\bigvee_{\sigma\in Sn}\left\{\left(D^{\alpha}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\alpha}_{n\sigma(n)}\right),\left(D^{\beta}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\beta}_{n\sigma(n)}\right),\left(D^{\gamma}_{1_{\sigma(1)}}\wedge\ldots\wedge D^{\gamma}_{n\sigma(n)}\right),\right\}]$$

$=|D^{C}_{P}|_{P}$. L.H.S=R.H.S
Hence $\left|D^{C}_{P}\odot_{P}D^{C}_{Q}\right|=|D^{C}_{P}|_{P}\odot_{P}|D^{C}_{Q}|_{P}$

THEOREM 4.4.
Let $D^{C}_{P}=\left(<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>\right)=[p_{ij}]$
and $D^{C}_{Q}=\left(<\left[D^{\underline{\mu}}_{Q_{ij}},D^{\overline{\mu}}_{Q_{ij}}\right],D^{\mu}_{Q_{ij}}>,<\left[D^{\underline{\nu}}_{Q_{ij}},D^{\overline{\nu}}_{P_{ij}}\right],D^{\nu}_{Q_{ij}}>,<\left[D^{\underline{\varsigma}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}\right],D^{\varsigma}_{Q_{ij}}>\right)=[q_{ij}]$ all $\in CPFSSM_{(mxn)}$,
then $\left|D^{C}_{P}\odot_{R}D^{C}_{Q}\right|=|D^{C}_{P}|_{R}\odot_{R}|D^{C}_{Q}|_{R}$

Proof:
L.H.S=$\left|D^{C}_{P}\odot_{R}D^{C}_{Q}\right|$
$\Rightarrow[\left(<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>\right)\odot_{R}$
$\left(<\left[D^{\underline{\mu}}_{Q_{ij}},D^{\overline{\mu}}_{Q_{ij}}\right],D^{\mu}_{Q_{ij}}>,<\left[D^{\underline{\nu}}_{Q_{ij}},D^{\overline{\nu}}_{P_{ij}}\right],D^{\nu}_{Q_{ij}}>,<\left[D^{\underline{\varsigma}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}\right],D^{\varsigma}_{Q_{ij}}>\right)$
Since $D^{C}_{P}\subseteq_{R}D^{C}_{Q}\leftrightarrow D^{\overline{\alpha}}_{P_{ij}}\leq D^{\overline{\mu}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\leq D^{\overline{\nu}}_{P_{ij}},\ D^{\overline{\gamma}}_{P_{ij}}\leq D^{\overline{\varsigma}}_{P_{ij}},\ D^{\underline{\alpha}}_{P_{ij}}\leq D^{\underline{\mu}}_{Q_{ij}},D^{\underline{\beta}}_{P_{ij}}\leq D^{\underline{\nu}}_{Q_{ij}},$
$D^{\underline{\gamma}}_{P_{ij}}\leq D^{\underline{\varsigma}}_{Q_{ij}}$, and $D^{\alpha}_{p_{ij}}\geq D^{\mu}_{Q_{ij}},D^{\beta}_{p_{ij}}\geq D^{\nu}_{Q_{ij}},D^{\gamma}_{p_{ij}}\geq D^{\varsigma}_{Q_{ij}}$, for all i ,j by Definition 3.2.
$\left|D^{C}_{P}\odot_{R}D^{C}_{Q}\right|=<\left[min\left\{D^{\underline{\alpha}}_{P_{ij}},D^{\underline{\beta}}_{P_{ij}},D^{\underline{\gamma}}_{P_{ij}}>,<D^{\underline{\mu}}_{Q_{ij}},D^{\underline{\nu}}_{Q_{ij}},D^{\underline{\varsigma}}_{Q_{ij}}>\right\},min\left\{D^{\overline{\alpha}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}>,<\right.\right.$
$\left.\left.D^{\overline{\mu}}_{Q_{ij}},D^{\overline{\nu}}_{Q_{ij}},D^{\overline{\varsigma}}_{Q_{ij}}>\right\},max\left\{D^{\alpha}_{P_{ij}},D^{\beta}_{P_{ij}},D^{\gamma}_{P_{ij}}>,<D^{\mu}_{Q_{ij}},D^{\nu}_{P_{ij}},D^{\varsigma}_{P_{ij}}>\right\}\right]>for\ all\ i,j.$
$\Rightarrow\left(<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>\right)$
It is that, $\left|D^{C}_{P}\odot_{R}D^{C}_{Q}\right|=|D^{C}_{P}|_{R}$ by Definition 3.4
$|D^{C}_{P}|_{R}=\left(<\left[D^{\underline{\alpha}}_{P_{ij}},D^{\overline{\alpha}}_{P_{ij}}\right],D^{\alpha}_{p_{ij}}>,<\left[D^{\underline{\beta}}_{P_{ij}},D^{\overline{\beta}}_{P_{ij}}\right],D^{\beta}_{p_{ij}}>,<\left[D^{\underline{\gamma}}_{P_{ij}},D^{\overline{\gamma}}_{P_{ij}}\right],D^{\gamma}_{p_{ij}}>\right)$ is defined as,

$|D_P^C|_R = [\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\underline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\alpha}}),(D_{1_{\sigma(1)}}^{\underline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\beta}}),(D_{1_{\sigma(1)}}^{\underline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\gamma}}),\},$
$\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\overline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\alpha}}),(D_{1_{\sigma(1)}}^{\overline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\beta}}),(D_{1_{\sigma(1)}}^{\overline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\gamma}}),\},$
$\bigwedge_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\alpha}\vee\ldots\vee D_{n\sigma(n)}^{\alpha}),(D_{1_{\sigma(1)}}^{\beta}\vee\ldots\vee D_{n\sigma(n)}^{\beta}),(D_{1_{\sigma(1)}}^{\gamma}\vee\ldots\vee D_{n\sigma(n)}^{\gamma}),\}]$
$|D_Q^C|_R = (<[D_{Q_{ij}}^{\underline{\mu}},D_{Q_{ij}}^{\overline{\mu}}],D_{Q_{ij}}^{\mu}>,<[D_{Q_{ij}}^{\underline{\nu}},D_{P_{ij}}^{\overline{\nu}}],D_{Q_{ij}}^{\nu}>,<[D_{Q_{ij}}^{\underline{\varsigma}},D_{Q_{ij}}^{\overline{\varsigma}}],D_{Q_{ij}}^{\varsigma}>)$ is defined as,

$|D_Q^C|_P = [\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\underline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\alpha}}),(D_{1_{\sigma(1)}}^{\underline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\beta}}),(D_{1_{\sigma(1)}}^{\underline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\gamma}}),\},$
$\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\overline{\mu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\mu}}),(D_{1_{\sigma(1)}}^{\overline{\nu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\nu}}),(D_{1_{\sigma(1)}}^{\overline{\varsigma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\varsigma}}),\},$
$\bigwedge_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\mu}\vee\ldots\vee D_{n\sigma(n)}^{\mu}),(D_{1_{\sigma(1)}}^{\nu}\vee\ldots\vee D_{n\sigma(n)}^{\nu}),(D_{1_{\sigma(1)}}^{\varsigma}\vee\ldots\vee D_{n\sigma(n)}^{\varsigma}),\}]$
Then $|D_P^C|_R \odot_R |D_Q^C|_R = |<[D_{P_{ij}}^{\underline{\alpha}},D_{P_{ij}}^{\overline{\alpha}}],D_{p_{ij}}^{\alpha}>,<[D_{P_{ij}}^{\underline{\beta}},D_{P_{ij}}^{\overline{\beta}}],D_{p_{ij}}^{\beta}>,<[D_{P_{ij}}^{\underline{\gamma}},D_{P_{ij}}^{\overline{\gamma}}],D_{p_{ij}}^{\gamma}>|_R \odot_R |<$
$[D_{Q_{ij}}^{\underline{\mu}},D_{Q_{ij}}^{\overline{\mu}}],D_{Q_{ij}}^{\mu}>,<[D_{Q_{ij}}^{\underline{\nu}},D_{P_{ij}}^{\overline{\nu}}],D_{Q_{ij}}^{\nu}>,<[D_{Q_{ij}}^{\underline{\varsigma}},D_{Q_{ij}}^{\overline{\varsigma}}],D_{Q_{ij}}^{\varsigma}>|_R$
$\Rightarrow [\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\underline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\alpha}}),(D_{1_{\sigma(1)}}^{\underline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\beta}}),(D_{1_{\sigma(1)}}^{\underline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\gamma}}),\},$
$\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\overline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\alpha}}),(D_{1_{\sigma(1)}}^{\overline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\beta}}),(D_{1_{\sigma(1)}}^{\overline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\gamma}}),\},$
$\bigwedge_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\alpha}\vee\ldots\vee D_{n\sigma(n)}^{\alpha}),(D_{1_{\sigma(1)}}^{\beta}\vee\ldots\vee D_{n\sigma(n)}^{\beta}),(D_{1_{\sigma(1)}}^{\gamma}\vee\ldots\vee D_{n\sigma(n)}^{\gamma}),\}] \odot_R$
$[\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\underline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\alpha}}),(D_{1_{\sigma(1)}}^{\underline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\beta}}),(D_{1_{\sigma(1)}}^{\underline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\gamma}}),\},$
$\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\overline{\mu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\mu}}),(D_{1_{\sigma(1)}}^{\overline{\nu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\nu}}),(D_{1_{\sigma(1)}}^{\overline{\varsigma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\varsigma}}),\},$
$\bigwedge_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\mu}\vee\ldots\vee D_{n\sigma(n)}^{\mu}),(D_{1_{\sigma(1)}}^{\nu}\vee\ldots\vee D_{n\sigma(n)}^{\nu}),(D_{1_{\sigma(1)}}^{\varsigma}\vee\ldots\vee D_{n\sigma(n)}^{\varsigma}),\}]$

By the Definition 3.2
$\Rightarrow \{[min \bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\underline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\alpha}}),(D_{1_{\sigma(1)}}^{\underline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\beta}}),(D_{1_{\sigma(1)}}^{\underline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\gamma}})\},$
$\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\underline{\mu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\mu}}),(D_{1_{\sigma(1)}}^{\underline{\nu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\nu}}),(D_{1_{\sigma(1)}}^{\underline{\varsigma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\varsigma}}),\}]$
$[\min \bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\overline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\alpha}}),(D_{1_{\sigma(1)}}^{\overline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\beta}}),(D_{1_{\sigma(1)}}^{\overline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\gamma}})\},$
$\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\overline{\mu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\mu}}),(D_{1_{\sigma(1)}}^{\overline{\nu}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\nu}}),(D_{1_{\sigma(1)}}^{\overline{\varsigma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\varsigma}}),\}]$
$[\max \bigwedge_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\alpha}\vee\ldots\vee D_{n\sigma(n)}^{\alpha}),(D_{1_{\sigma(1)}}^{\beta}\vee\ldots\vee D_{n\sigma(n)}^{\beta}),(D_{1_{\sigma(1)}}^{\gamma}\vee\ldots\vee D_{n\sigma(n)}^{\gamma}),\},$
$\bigwedge_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\mu}\vee\ldots\vee D_{n\sigma(n)}^{\mu}),(D_{1_{\sigma(1)}}^{\nu}\vee\ldots\vee D_{n\sigma(n)}^{\nu}),(D_{1_{\sigma(1)}}^{\varsigma}\vee\ldots\vee D_{n\sigma(n)}^{\varsigma}),\}]\}$
$\Rightarrow |D_P^C|_R = [\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\underline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\alpha}}),(D_{1_{\sigma(1)}}^{\underline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\beta}}),(D_{1_{\sigma(1)}}^{\underline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\underline{\gamma}}),\},$
$\bigvee_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\overline{\alpha}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\alpha}}),(D_{1_{\sigma(1)}}^{\overline{\beta}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\beta}}),(D_{1_{\sigma(1)}}^{\overline{\gamma}}\wedge\ldots\wedge D_{n\sigma(n)}^{\overline{\gamma}}),\},$
$\bigwedge_{\sigma\in Sn}\{(D_{1_{\sigma(1)}}^{\alpha}\vee\ldots\vee D_{n\sigma(n)}^{\alpha}),(D_{1_{\sigma(1)}}^{\beta}\vee\ldots\vee D_{n\sigma(n)}^{\beta}),(D_{1_{\sigma(1)}}^{\gamma}\vee\ldots\vee D_{n\sigma(n)}^{\gamma}),\}]$

$|D_P^C|_R$. L.H.S=R.H.S
Hence $|D_P^C \odot_R D_Q^C| = |D_P^C|_R \odot_R |D_Q^C|_R$

IV. CONCLUSION:

In this document, we investigated some properties to the determinant theory of cubic picturefuzzy soft square matrices with suitable examples. In future, this can paves a new way for the advanced mathematical research to a new concept of the determinant theory of cubic Pythagorean fuzzy soft square matrices'

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