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Minimally doubled fermions in Fermi QCD parallel lattice calculations

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ABSTRACT : FermiQCD is a C++ library for fast development of parallel Lattice Quantum Field Theory computations. We have used this package in our previous works, for testing algorithms, programs scalability and lattice computations. In this work, we have added to the package another fermionic action: Boriçi - Creutz action, have calculated the light hadrons spectrum with this action and also have studied the scalability of the programs during parallel calculations. We present simulations for the light hadrons spectrum with the BC action in quenched QCD carried out on the Computational Physics Group's server in the Department of Physics, University of Tirana. Simulations are made with gauge action on 48^4,64^4 lattices for a total number of 500 SU(3) gauge configurations. The masses of pi and rho – meson, nucleon and delta baryons are computed for these conditions. Also, the scalability of the application in parallel is tested.

KEYWORDS : FermiQCD, Boriçi - Creutz fermions, parallel computing, scalability

I. INTRODUCTION

Quantum Chromodynamics (QCD) is the theory of strong interaction. It describes how quarks and gluons interact in the strong force regime. It's important to say that in the low energy regime, perturbative calculations don't have any values. That's why a non - perturbative approach was proposed by Wilson in 1974: Lattice QCD [15]. It is a lattice gauge theory formulated on a grid or lattice of points in space and time. When the size of the lattice is taken infinitely large and its sites infinitesimally close to each other, the continuum QCD is recovered [15]. Because the computational cost of numerical simulations can increase as the lattice spacing decreases, often the results are extrapolated to a = 0 by repeated calculations at different lattices spacing a. Anyway there is a problem that appear when naively we try to put fermions fields on a lattice. It consists in the appearance of other states, such that one ends up having 2^d fermionic particles (with d the number of discretized dimensions) for each original fermion. By the no-go theorem [12], the minimum number of species one can have on a lattice with chiral symmetry is two, that is what is known as a "minimally doubled" fermion. There are several formulation of minimally doubled fermions, but we will be focused on Boriçi - Creutz (BC) fermions [2, 5]. These fermions are strictly local while preserving exact chiral symmetry for a degenerate doublet of quark fields, but breaks the hypercubic symmetry [1]. The renormalization properties of the BC fermions at one loop in the perturbation theory have been investigated in reference [3, 4] and the free Dirac operator for Boriçi-Creutz fermions in momentum space is written as:

$$D_{BC}(p) = \sum_{\mu}[i\gamma_{\mu} \sin p_{\mu} + i\left(\Gamma - \gamma_{\mu}\right) \cos p_{\mu}] + i(c_3 - 2)\Gamma \qquad (1)$$

where c_3 is one of the counter terms added in the action by the authors of reference [4], the most relevant term that contributes in the corrected action. The first step we had to do, was to add this action in FermiQCD package.

Let's remember that FermiQCD [6,17] is a C++ library for fast development of parallel QFT applications. Some detailed features of it are listed below:

- fully C++ (uses exceptions, streams, templates, inheritance, etc…)
- top-down design
- includes linear algebra and statistical package
- multiple lattices and fields
- automatic parallelization
- parallel random number generator
- parallel field::save and field::load methods (inherited)
- gauge_field for arbitrary SU(n)

- fermi_field for arbitrary SU(n)
- staggered_field for arbitrary SU(n) and even ndim
- Wilson, Clover, Asqtad actions (in SSE2 for SU(3)) (un-isotropic)
- Domain Wall action
- Fermilab action for heavy quarks (all dim=6 operators)
- minimum residue, stabilized bi-conjugate and uml inverters
- reads UKQCD, CANOPY, MILC and serial data formats
- easy
- safe: no need to use pointers
- flexible: can define your own fields and libraries by inheritance [6,17]

II. MATERIALS AND METHODS

After we have implemented Boriçi - Creutz fermions in FermiQCD parallel package, we have followed the below described procedure. The parallel calculations are carried out on the Computational Physics Group's server in the Department of Physics, University of Tirana, using FermiQCD, with Boriçi - Creutz operator. Quenched gauge configurations are generated with the Wilson gauge action at β = 5.7, 5.85 on lattices of size 48^4 and 64^4. For 500 configurations at each of the two lattice sizes, BC quark propagators are calculated for a single point source and all color - spin combinations. Propagators are calculated for five values of the hopping parameter κ: 0.138, 0.140, 0.142, 0.144, 0.147, corresponding to five lattice mass quarks. For the free theory the quark mass is given in terms of the lattice parameters κ and r as:

$$m_q a = \frac{1}{2\kappa} - 4r \equiv \frac{1}{2\kappa} - \frac{1}{2\kappa_c} \qquad (2)$$

with a zero at $\kappa = \kappa_c \equiv 1/8r$ .For the interacting theory ($U_\mu(x) \neq 1$)we will continue to define $m_q a = 1/2\kappa - 1/2\kappa_c$ with the proviso that κ_c depends on a. To compute masses of the hadrons we define their interpolating operators (currents) :

$$\begin{aligned} O_\pi &= \bar{\psi}\gamma_5\psi \\ O_{\rho,k} &= \bar{\psi}\gamma_5\gamma_k\psi \\ O_{N,\alpha} &= (\bar{\psi} \wedge C\gamma_5\psi)\psi_\alpha \\ O_{\Delta,k,\alpha} &= (\bar{\psi} \wedge C\gamma_k\psi)\psi_\alpha \end{aligned} \qquad (3)$$

where the wedge (vector) product is carried out in color space. The charge conjugation matrix C is defined by: $C\gamma_\mu C = -\gamma_\mu^T$ and we have chosen $C = i\gamma_2\gamma_4$. Restoring the space indices, hadron propagators are given by the correlation functions **$S_{ij} := O_i O_j^*$,** where O_i is a generic current located at site i. The Euclidean hadrons propagators on the lattice with periodic boundary conditions, for zero momentum, can be written as:

$$S_{t,t_0} \simeq \frac{1}{2} c_1 \cosh am_1 (t - t_0 - L/2) + \frac{1}{2} c_2 \cosh am_2 (t - t_0 - L/2) \qquad (4)$$

where L is the lattice extension in time direction, m_1 is the mass of the ground state of the particle and m_2 the mass of its first exited state. [14] To compute the hadrons propagators we have used static point sources located at t_0 = (0, 0, 0, 1) on 500 configurations. As a solver, we have used the Stabilized Biconjugate Gradient method, BiCGStab.

$$\frac{S_{t+1,1}}{S_{t,1}} = \frac{\cosh am(t - L/2)}{\cosh am(t - 1 - L/2)}, t = 1,........L \qquad (5)$$

for mesons and

$$\frac{S_{t+1,1}}{S_{t,1}} = \frac{\sinh am(t - L/2)}{\sinh am(t - 1 - L/2)}, t = 1,........L \qquad (6)$$

for baryons. Point splitting method is used for defining each quark field [14] and we have calculated the masses in two different directions: according to one edge and according the diagonal of the hyper cube.

Errors are estimated using Jackknife method. The existing codes of FermiQCD are improved and new codes need for these calculations are written.

III. RESULTS

In the first figure we present the result for the pi-meson, while in the second one are presented the masses of rho meson, nucleon and delta baryon in two different directions, using BC corrected action.

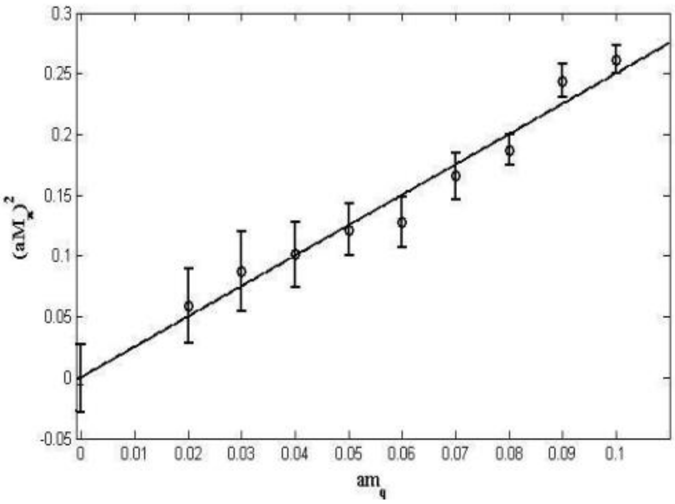

Fig 1. Pion mass using BC action. According to the chiral theory the square mass goes to zero at a vanishing quark mass

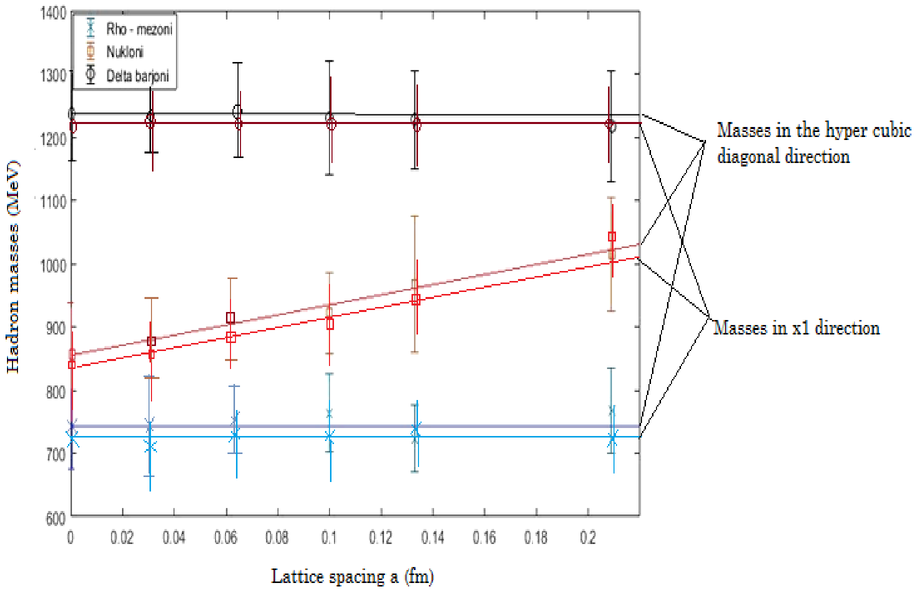

Fig 2. Masses of the rho meson, nucleon and delta baryon using BC action in two different directions

As it can be seen by the results there is a slight difference between the the same hadron masses calculated in different direction, a detail which can be expected, due to the hypercubic symmetry partial restauration. Anyway the masses in the continuum limit go to the values we expect.

Let's see what happen with the execution time of our application when we increase the number of processors used. In Fig. 3, we illustrate the execution times of the quenched QCD application on the Physics Group's server in the Department of Physics, University of Tirana for a 48^4 lattice.

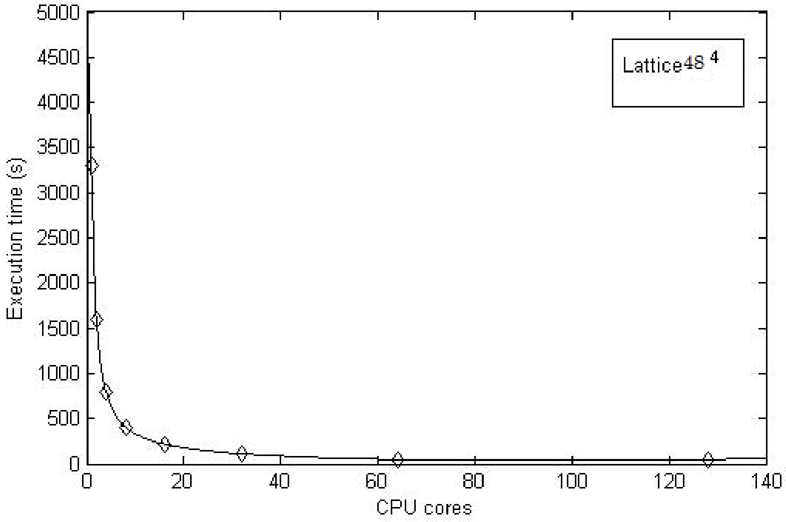

Fig 3. Execution time of the application for different number of cores

To elucidate the scalability of the LQCD application we calculate the speed - up $\mathbf{S = T_1/T_P}$, where T_1 and T_P is the execution time for single and number of P processors, respectively. Fig. 4 shows the obtained S on the Computational Physics Group's server in the Department of Physics .

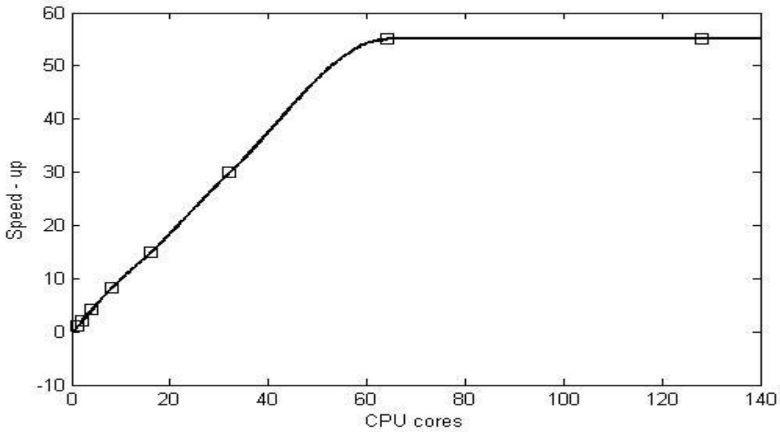

Fig 4. The speedup of the quenched QCD for different number of cores

As we see we have a good scalability of the application.

IV. CONCLUSIONS

FermiQCD libraries provide a tool for fast development of parallel QFT (quantum field theory) applications and our application implemented in FermiQCD seems to have a very good scalability. Implementing BC fermions to this package has its difficulties, because this action break the hypercubic symmetry and even when we implemte it with the corrected counterterms, we have to be accurate with the calculations in different directions. Anyway, The results taken by using it's gauge fields and the corrected Boriçi - Creutz action are in agreement at a level of 10%, with the numerical simulations done until now with other software-s and techniques. Sure, making spectroscopy with this kind of fermions isn't the so simple and has a lot of work to do.

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