IJMCER

International Journal of Multidisciplinary and Current Educational Research (IJMCER)
ISSN: 2581-7027 ||Volume|| 4 ||Issue|| 1 ||Pages 62-66 ||2021||

Baumol's Law application in Performing Arts: An overview of the mathematical model in relation to a state's overall Economy

[1,]StamatisGargalianos, [2,]Dimitra Tsiaka, [3,]Vasileios E. Papageorgiou
[1,]Department of Early Childhood Education, University of Western Macedonia, 53100 Florina, Greece
[2,]Principal in Secondary Education in the Prefecture of Fthiotida, 35100 Lamia, Greece
[3,]Department of Mathematics, Aristotle University of Thessaloniki, Thessaloniki 54124, Greece

ABSTRACT : This article is an attempt to associate Baumol's Law and the Performing Arts (PAs), based on mathematical modelling. In 1966, Americans Baumol and Bowen published for the first time an analysis regarding the financing of live entertainment/performing arts; as this was the first analysis in this field, and precisely because it sought to establish a mathematical-economic relationship between the needs of performing arts and State grants, it was called "Baumol's Law". This "law" provides a rough list of reasons for performing arts' inability to be self-financed; at the same time, it highlights the need to financially support institutions of any form, size or level via external bodies, such as the state, patrons of art, and prominent sponsors. A distinction is made between the anthropocentric and the machine sector; in the former, human labour is key, while the latter is dominated by machines and all their derivatives.

KEYWORDS: Economy, Baumol's Law, Performing Arts, Productivity, Wages, Theatre, Anthropocentric Sector, Growing Sector, Mathematics, State, Workforce, Labour, Overall Production, Productivity

I. INTRODUCTION

The "cornerstone" of Baumol's reasoning is none other than the basic fact that the hitherto classic sectors of the Economy (industry, commerce) have a huge advantage over the idiosyncrasy of performing arts: the possibility to replace human labour with technological innovations, such as machines and robots. Thus, in these sectors, the final product's price can be reduced, thanks to a gradual and smooth replacement of the workforce, which is attributable to the automation offered by technological advances (Gargalianos&Giannakopoulou, 2012: 34). On the contrary, sectors such as the Performing Arts (PA) rely -without the possibility of an alternative- solely on the human factor, with all the medium- and long-term implications this may have on the cost of the "product" that is being produced. Here, the development and use of new technologies does not bring any change -reduction- in the price of the final product, since, in PAs, human labour can only be replaced to a limited extent by new technological advances. In this sense, and according to the same reasoning, all economic activity around the world can be divided into two main categories, that of standard industry, the so-called "growing" sector, and that of performing arts, the so-called "non-growing" sector (List, 2017: 178). This verbal distinction between the two sectors is based -as mentioned above- on their ability, or lack thereof, to progressively increase productivity, hence profits, through the utilization of technological means.

Through the Baumol's Law, which allows the mathematical representation of the examined phenomenon, we underline the need of financial support in the field of performing arts. For the purposes of this analysis, we define the most important parameters that we encounter in the following mathematical presentation, such as$L$: indicating the size of the workforce; Y: the quantity of final production; t: time; r: the rate of productivity increase; W: the amount of worker wages; C: the total cost of production (Gargalianos, 1994: 129).

II. METHODOLOGY

The methodology we followed in order to write this article was the constant comparison between Baumol's Law and the needs of the Performing Arts (PAs).We were based for that on a mathematical model, consisting of logarithms, constants, variants and numerous equations. We knew that the development and use of new technologies does not bring any change -reduction- in the price of the final artistic product, since, in PAs, human labour can only be replaced to a limited extent by new technological advances. First, we took into consideration a mathematical-economic model which explains the relationship between the needs of performing arts and the State grants. Our mathematical analysis, in terms of methodology, is based on models and parameters such as L (size of the workforce), Y (quantity of final production), t (time of production), r (rate of productivity increase), W (worker wages), and C (total cost of production). Thanks to these models, and based on a distinction between the anthropocentric and the machine sector, we made the connection between the performing arts and the financial support of each government.In that way, and according to the same reasoning, we examined the economic activities around the world and we understood that they can be divided into two main categories, that of standard industry, the so-called "growing" sector, and that of performing arts, the so-called "non-growing" sector. We studied the distinction between the two sectors and we made reflexions on their ability, or lack thereof, to progressively increase productivity, hence profits, through the utilization of technological means. In that purpose we were based on the Baumol's Law, because it the mathematical representation of the examined phenomenon, we can underline the need of financial support in the field of performing arts. We assigned numbers to all important equations in this model, in order to facilitate our reflexion, as well as its readability for readers, who must take into consideration that this mathematical model is rather complex.

Baumol's Law in Performing Arts: In order to simplify the continuity of their calculations, Baumol and Bowen assume that the costs of both sectors are fixed and that both sectors' product purchase price by the public depends on labour costs, i.e. wages (Moreau, 2004: 862). This implies that families spend a fixed portion of their income on the purchase of anthropocentric sector products -entertainment, other cultural services- because each increase in ticket prices corresponds to a respective increase in worker wages(Rothbard, 2009: 892). These last considerations can be represented using parameters a and $b$as follows

$$P1 = \alpha \cdot C1 \quad (1)$$

$$P2 = b \cdot C2 \quad (2)$$

Assuming that the ratio of labour quantities used in both sectors 1 and 2 remains constant, we obtain $\frac{L_1}{L_2} = K_1$ (constant). Then, it's certain that the relationship $\frac{Y_1}{Y_2}$ equals to $\frac{a \cdot L_1}{b \cdot L_2\,(1+r)^t}$, because $Y_1 = a \cdot L_1$ and$Y_2 = b \cdot L_2\,(1+r)^t$.

If we then replace $\frac{L_1}{L_2}$ with K_1 we get$\frac{Y_1}{Y_2} = \frac{a \cdot K_1}{b \cdot (1+r)^t}$.

The limit of$\frac{Y_1}{Y_2}$ when $t \to \infty$ is equal to 0, because: $\lim_{t\to\infty} b(1+r)^t = \infty$

so, we get$\lim_{t\to\infty} \frac{a \cdot K_1}{b \cdot (1+r)^t} = 0$.

This means that, in along-theoretically infinite- time period, the production Y_2 (industrial sector) becomes infinite, while the production Y_1-anthropocentric sector- remains constant, i.e. in the face of maximum production in Y_2, Y_1could be deemed as zero.How can we react when faced with such a situation, which -let us emphasise- will eventually lead to the absolute decline of the anthropocentric sector? It is obvious that the relationship $\frac{Y_1}{Y_2}$must be kept constant. This can be achieved either through a high demand for cultural products -namely, that the public consumes only art products, which is unrealistic- or through an intervention by public authorities (Baumol& Bowen, 1966: 178). Therefore, we assume that$\frac{Y_1}{Y_2} = K_2$ (constant), thus, we get $Y_1 = Y_2 \cdot K_2$.

Assuming that the total workforce is L, we have

$$L = L_1 + L_2 \Rightarrow L_2 = L - L_1 \quad (3)$$

The workforce $L_1$of the non-growing sector is equivalent to

$$L_1 = \frac{Y_1}{a} = \frac{Y_2 \cdot K_2}{a} = \frac{b \cdot L_2 \cdot (1+r)^t \cdot K_2}{a} \quad (4)$$

Considering equation (3), this workforce becomes

$$L_1 = \frac{b \cdot (1+r)^t \cdot K_2 \cdot (L - L_1)}{a} \Rightarrow$$

$$L_1 = \frac{b \cdot (1+r)^t \cdot K_2 \cdot L}{a} - \frac{L_1 \cdot b(1+r)^t \cdot K_2}{a} \Rightarrow$$

$$L_1 + \frac{L_1 \cdot b\,(1+r)^t \cdot K_2}{a} = \frac{b\,(1+r)^t \cdot K_2 \cdot L}{a} \Rightarrow$$

$$L_1 \cdot \left(1 + \frac{b\,(1+r)^t \cdot K_2}{a}\right) = \frac{b\,(1+r)^t \cdot K_2 \cdot L}{a} \Rightarrow$$

$$L_1 = \frac{\frac{b\,(1+r)^t \cdot K_2 \cdot L}{a}}{1 + \frac{b\,(1+r)^t \cdot K_2}{a}} \tag{5}$$

Considering that K_2 is a constant, $\frac{b}{a} K_2$ can be replaced by K_3 which is also a constant. Therefore, we have

$$L_1 = \frac{K_3 \cdot (1+r)^t \cdot L}{1 + (1+r)^t \cdot K_3} \tag{6}$$

Regarding L_2, and according to the same reasoning, we will have

$$L_2 = L - L_1 = L - \frac{K_3 \cdot (1+r)^t \cdot L}{1 + (1+r)^t \cdot K_3} \Rightarrow$$

$$L_2 = \frac{L\,(1 + (1+r)^t \cdot K_3) - K_3 \cdot (1+r)^t \cdot L}{1 + (1+r)^t \cdot K_3} \Rightarrow$$

$$L_2 = \frac{L}{1 + (1+r)^t \cdot K_3} \tag{7}$$

After mathematically calculating the value of L_1 and L_2, we need to know what result they produce in infinite time. Therefore

$$\lim_{t \to \infty} L_1 = \lim_{t \to \infty} \frac{K_3 \cdot L \cdot (1+r)^t}{1 + (1+r)^t \cdot K_3}$$

$$= \lim_{t \to \infty} \frac{\frac{K_3 \cdot L\,(1+r)^t}{(1+r)^t}}{\frac{1}{(1+r)^t} + \frac{(1+r)^t \cdot K_3}{(1+r)^t}}$$

$$= \lim_{t \to \infty} \frac{L \cdot K_3}{\frac{1}{(1+r)^t} + K_3} = \frac{L \cdot K_3}{K_3} = L$$

Namely,

$$\lim_{t \to \infty} L_1 = L. \tag{8}$$

Regarding the limit of L_2, we will have

$$\lim_{t\to\infty} L_2 = \lim_{t\to\infty} \frac{L + L \cdot K_3 \cdot (1+r)^t - L \cdot K_3 \cdot (1+r)^t}{1 + K_3 \cdot (1+r)^t}$$

$$= \lim_{t\to\infty} \frac{\frac{L}{(1+r)^t}}{\frac{1}{(1+r)^t} + \frac{K_3\,(1+r)^t}{(1+r)^t}}$$

$$= \lim_{t\to\infty} \frac{\frac{L}{(1+r)^t}}{\frac{1}{(1+r)^t} + K_3} = 0$$

As a result, we have

$$\lim_{t\to\infty} L_2 = 0 \qquad (9)$$

These two mathematical results (8) and (9) imply that in an infinite amount of time and in order to maintain the relationship $\frac{Y_1}{Y_2}$ constant, and therefore> 0, all labour force -that is, all workers- will eventually end up working in the non-growing sector, as the workforce in the second sector will be constantly decreasing as we move towards more and more future time intervals. Under these conditions, let us examine what will happen to the overall production. It is known, initially, that

$$Y_1 = a \cdot L_1$$

and that

$$Y_2 = b \cdot (1+r)^t \cdot L_2$$

Based on the fact that$Y = Y_1 + Y_2$, represents the overall quantity of final production,Y becomes

$$Y = a \cdot L_1 + b \cdot (1+r)^t \cdot L_2 \quad (10)$$

But L_1 and L_2 already correspond to certain values (6) and (7). Therefore

$$Y = a \; \frac{K_3\,(1+r)^t \cdot L}{1 + (1+r)^t \cdot K_3} + b\,(1+r)^t \frac{L}{1 + K_3\,(1+r)^t}$$

$$= \frac{a \cdot K_3 \cdot L \cdot (1+r)^t + b\,(1+r)^t\,L}{1 + K_3\,(1+r)^t}$$

$$= \frac{a \cdot K_3 \cdot L \cdot (1+r)^t + b \cdot L \cdot (1+r)^t}{1 + K_3 \cdot (1+r)^t} = \frac{(1+r)^t \cdot L \cdot (a\,K_3 + b)}{1 + K_3\,(1+r)^t}$$

Because L and K_3 are constant, we can get$L(a \cdot K_3 + b) = K_4$, where K_4 is a new constant that will be inserted into the previous equation, in order to have

$$Y = \frac{(1+r)^t}{1 + K_3 \cdot (1+r)^t} K_4 \quad (11)$$

Based on this equation, we can obtain the growth rate of the overall economy. For this reason, we must produce the derivative of $Y$with respect to t. We utilize the logarithm of Yaiming to examine the evolution of the corresponding growth rate. Moreover, by taking the derivative of the logarithmic value, we culminate in

$$\log Y = \log \frac{(1+r)^t}{1+ K_3\,(1+r)^t} K_4$$

$$\log Y = \log\ (1+r)^t \cdot K_4 - \log (1+ K_3\,(1+r)^t)$$

$$= \log\ (1+r)^t + \log\ K_4 - \log\ (1+ K_3(1+r)^t)$$

$$= t\ \cdot \log\ (1+r) + \log K_4 - \log\ (1+ K_3(1+r)^t)$$

$$(\log Y)' = (t\ \cdot \log(1+r))' + (\log\ K_4)' - \log\ (1+ K_3\,(1+r)^t)' \Rightarrow$$

$$(\log Y)' = \log(1+r) + 0 - \frac{K_3\,(1+r)^t \cdot \log\ (1+r)}{1+ K_3\,(1+r)^t} \Rightarrow$$

$$(\log Y)' = \frac{\log(1+r) \cdot (1+K_3(1+r)^t) - K_3\,(1+r)^t \cdot \log(1+r)}{1+ K_3\,(1+r)^t} \Rightarrow$$

$$(\log Y)' = \frac{\log(1+r)}{1+ K_3\,(1+r)^t} \qquad (12)$$

The above equation demonstrates the growth rate of the Economy which, in aninfinite time, becomes

$$\lim_{t\to\infty} (logY)' = 0$$

because $\lim_{t\to\infty}(1 + K_3(1+r)^t) = \infty$.

This implies that in a limited time t and in order to maintain the relationship $\frac{Y_1}{Y_2}$constant, as well as greater than 0 -namely: if we do not want the PAs to gradually disappear, the rate of economic growth will eventually becomezero (Rothbard, 2009: 505).This fact highlights the emergence of significant negative effects on the sector of performance arts, due to its constant downgrading in terms of funding. Although, the rapid development of the technological sector leading to the marginalization of the arts sector may lead to more general economic growth, but we must never forget the varied benefits that the arts sector offers to the psychosynthesis of society.

II. DISCUSSION - CONCLUSIONS

The above mathematical analysis was based on models and parameters such as L (size of the workforce), Y (quantity of final production), t (time of production), r (rate of productivity increase), W (worker wages), and C (totalcostof production). Thanks to these models,and based on a distinction between the anthropocentric and the machine sector, we demonstrated the main causes of performing arts' inability to self-finance, as well as the need to have constant financial support provided to respective institutions of any size, form or level by the government or other bodies.The most significant difference between the two sectors lies in the fact that, in the non-growing sector, an increase in worker wages does not imply a corresponding increase in productivity; on the contrary, in the growing sector, always according to the analysed mathematical equations, an increase in worker wages is combined with a rise in productivity, even if the latter is due to machines. It should also be noted that this second increase leads to a relative stability of final product prices in that sector. Thus, we conclude that PAs, in contrast to the production industries of the growing sector, have only three "sources" of funding: I) the public, II) the state, III) the sponsors. However, these three "sources" do not always support cultural institutions to the desired degree; consequently, the cultural products that are generated are not consumed, thus confirming Baumol's law.

REFERENCES

1. Barr, N. (2020). *Economics of the Welfare State*. Oxford: Oxford University Press.
2. Baumol, W., Bowen, W. (1966). *Performing Arts: The economic dilemma*. New York: The Twentieth Century Fund.
3. Baumol, W. (1967). *Performing Arts, the permanent crisis*. New York: Business Horizons Review.
4. Gargalianos, S. (1994). Gestion d' Institutions Culturelles - L' application de la Loi de Baumol dans le cas du théâtre grec subventionné. Thèse de 3e Cycle. Université PARIS-IX - Dauphine.
5. Gargalianos, S., Giannakopoulou, I. (2012). *Theatre Management-Marketing*. Thessaloniki: Romi Publications.
6. List, F. (2017). *National system of political economy: all four books and the appendices in a single volume*. California: Createspace.
7. Moreau, F. (2004). The role of the state in evolutionary economics. Cambridge Journal of Economics, v. 28,No. 6.
8. Rothbard, M. (2009). *Man, Economy and the State with Power and Market*. Martinsburg: Ludwig von Mises Institute.