International Journal of Multidisciplinary and Current Educational Research (IJMCER)
ISSN: 2581-7027 ||Volume|| 4 ||Issue|| 6 ||Pages 32-45 ||2022||

Planning of the production activities of a manufacturing system in the context of a closed-loop supply chain.

Kibouka G. Richard[1], Mandatsy Moungomo J. Brice[2], Ndong Mezui J.M Lauhic[3]
[1,2,3,] *Department of Mechanical Engineering of ENSET, Systems Technology Research Laboratory*
210 Avenue des Grandes Écoles, 3989 Libreville, Gabon

ABSTRACT : This article presents a problem of planning production activities, equipment maintenance, supply and industrial logistics in a closed loop, for a production system evolving in a stochastic context. The hybrid production/reuse system considered in this study consists of two machines. These machines, designated M_1 and M_2 are assigned respectively to production (manufactured) and reconditioning (remanufactured). We therefore consider a hybrid system based on the recovery of a single type of products where requests can be met from an inventory of finished products. This inventory can be replenished by the production and repackaging of end-of-life products (used and returned). Another inventory is available for the preservation of end-of-life products returned prior to the repackaging process. Returned products may be repackaged, disposed of or kept in stock for later repackaging. The objective of the study is to develop a dynamic stochastic model to optimize the performance of a closed-loop logistics network. The reverse logistics network considered will have to establish a relationship between the market that releases used products and the market for "new" products. The main contribution of this study is to develop production policies in the different modes of the system. We have developed new optimality conditions in terms of modified Hamilton-Jacobi-Bellman equations (HJB) and recursive numerical methods applied to solve such equations. A numerical example and sensitivity analysis made it possible to determine the structure of the optimal policies and to show the usefulness and robustness of the results obtained.

KEYWORDS : Stochastic process, reverse logistics, numerical methods, quality, production/reuse.

I. INTRODUCTION

Manufacturing industries are faced with the challenges of optimizing their overall production supply chain. Production planning problems become more complex when environmental constraints require the optimization of manufacturing processes and the reuse, in manufacturing, of parts returned by consumers after use (reverse logistics). Compared to a situation where customer demand is only satisfied by the parts in the direct line (production from raw materials), the simultaneous control of production and *reuse (remanufacturing*) is very complex (Kiesmüller, [1]). Reuse problems are found in the fields of mechanical parts manufacturing, aluminum processing, vehicle and aircraft assembly lines, computers, photocopiers, *etc*. So, it's a question of how to plan production in a way that meets demand and minimizes total cost. Reverse logistics has several synonyms, although there are similarities between the terms, but they do not say the same thing (Lambert and Riopel, [2]). Stock [3] uses terms like recycling, waste destruction, and hazardous materials management to define reverse logistics.

Later, Fleischmann [4] proposed a new definition of reverse logistics as the process of planning, executing and controlling the flow of collected products (returns) in a supply chain (production) with the aim of putting them back on the market. Lambert and Riopel [2] mention that reverse logistics is the process of planning, implementing, and controlling the efficiency, profitability of raw materials, work-in-progress, finished products, and relevant information from the point of use to the point of origin in order to take back or generate value or to dispose of it in the right way while ensuring an efficient and environmental use of the resources put in place. work. While for Bennekrouf *and al.* [5], reverse logistics includes several profiles, namely: the return of products (following the non-satisfaction of a criterion), the reuse of certain products (such as packaging and containers), reprocessing (*remanufacture*) and cannibalization (dismantling of a product to reuse its parts). The latter depends on the quality index of the recovered products. Carter and Ellram [6] present reverse distribution as return, the counter-current movement of a product or material resulting from reuse, recycling or disposal.

According to Abdessalem *and al.* [7], reusing a product means that the product is used immediately in the same or another context, following a minor additional operation such as cleaning, maintenance. It can also mean the reuse of the parts that compose it as spare parts or raw materials. For Abdessalem *and al.* [7], recycling is defined as the action of collecting and disassembling a product at the end of its life cycle for material recovery. Remanufacturing *as* a process of disassembling used products, inspecting, repairing/replacing components and using them to manufacture a new product The reverse supply chain is characterized by a series of activities, according to (Bennekrouf *and al.* [5], the objective is the recovery of end-of-life products or components. With regard to the diversity of parameters, the volume of data and the different levels of decision-making (Hajji, [8]) shows that there is no universal approach to modelling. But for Min and Zhou [9], there are four approaches to Modeling namely; deterministic, stochastic, hybrid and models based on information technology. Hybrid production/reuse systems have expanded over the last thirty years with respect for the environment and environmental legislation (Mahadevan *and al.*, [10]. Reverse logistics establishes the relationship between the market for used and returned products and the market for new products. When the two markets coincide, we speak of a closed-loop network, if not an open loop (Salema *and al.*, [11]).

Several authors have worked in the context of hybrid production/reuse systems. Van der Laan and Salomon [12]). (1997) studied the *push-disposal* strategy, which consists of destroying all collected products when the stock level of finished products has reached the set threshold. They also studied the principle of *pull-disposal* which consists of destroying all collected products when the stock of returns has already reached the set threshold. They demonstrated that when the flow of returned products is less than the flow of demand, the total cost is less than the total cost of a system that does not account for destroyed returns. In Kiesmüller's approach [1], the *remanufacturing* process is only initiated when one wants to satisfy a demand (*Pull Policy* principle). Mahadevan *and al.* [10]) have studied the principle of P*ush Policy* according to which all returned items are directly *remanufactured*; therefore, no stock of returns.For (Gershwin, [13]), the system consists of two production centers subject to random phenomena: breakdowns, repairs, returns and variations in demand. According to (Van der Laan and Salomon, [12]), customer demand is satisfied by the stock of finished products. The return of products is composed of products at the end of their life cycle or already used. Returns that do not meet *remanufacturing* standards are not stored; they are destroyed. As extensions, the work of Gharbi *and* al [14], Berthaut *and al.* [15] and Pellerin *and al.* [16] have planned the production of a *remanufacturing* system that incorporates the case of unavailability of replacement parts. They assumed that the production system meets demand.

These authors framed their problem as a multi-level ordering policy based on critical stock thresholds. Their main maintenance and *remanufacturing* planning objective were to keep the stock of finished products at the optimal threshold. However, the authors only deal with *remanufacturing* without taking into account the direct line, i.e., *manufacturing*. The systematic analysis of the similarities and differences between the manufacturing, *remanufacturing* and repair process was done by Tongzhu *and al.* [17]. Kenne *and al.* [18] addressed production planning of a hybrid *manufacturing/remanufacturing* system in a closed-loop reverse logistics network. The objective was to propose a *manufacturing* and *remanufacturing* policy that minimizes the costs of stocking finished products and shortages. Following the analysis of the previous section, it emerges that the article by Kenne *et al.* [18] represents the first attempt in the study of hybrid *manufacturing/remanufacturing* systems with machines subject to random breakdowns and repairs.

The rest of the paper is organized as follows. In Section 2, we present the notations used and the assumptions of the model. Section 3 presents a formulation of the problem and the technical background. Section 4 is devoted to the development of the optimality conditions in the form of Hamilton-Jacobi-Bellman (HJB) equations and their numerical solutions. An illustrative example and results analysis are presented in Section 5. A sensitivity analysis and a comparative study are presented in Section 6 and 7, and the paper is concluded in Section 8.

II. NOTATION AND ASSUMPTION

This section presents the notations and assumptions used throughout this article. Table 1 highlights the notations used in this article.

Table 1

Notation	Désignation
u_1 (.)	Total production rate of Machine 1

u_2 (.)	Total production rate of Machine 2
U_1max	Max production rate on machine 1
U_2max	Max production rate on machine 2
d	Request for finished products
$\propto(t)$	Stochastic process describing system dynamics
$x_1(t)$	Stock of finished products at time t
$x_2(t)$	Stock of products returned at time t
q_{ij}	Transition rate from mode i to mode j
$MTTF_1$	Average uptime of Machine 1
$MTTR_1$	Machine 1 repair time
$MTTF_2$	Average uptime of Machine 2
$MTTR_2$	Machine 2 repair time
C_1^+	Product inventory cost 1
C_1^-	Shortage cost (Out of stock) of product 1
C_2	Cost of inventorying returned products
ρ	Discount rate

The system studied incorporates the following aspects:

1. The breakdown and repair rates of M1 and M2 machines are assumed to be constant (Not dependent on production).
2. Inventory costs corresponding to those produced for a positive inventory.
3. Shortage costs correspond to those associated with negative inventory.
4. The production rate of the production machine (M1) is higher than that of the reconditioning machine (M2).
5. The reconditioning machine alone cannot meet the demand.
6. Demand can be met by new products (manufactured or remanufactured).
7. The production machine transforms the raw material into finished products (new products).
8. The reconditioning machine transforms the returned products into new products.
9. It is permissible to produce to make up for unmet demands.
10. Returned product stock cannot be subject to shortages.

III. PROBLEM FORMULATION

The manufacturing system considered represents a common problem in the mining industry. The system consists of two machines designated M_1 and M_2 which are assigned respectively to production (manufactured) and reconditioning (remanufactured). These machines are subject to breakdowns and random repair actions that can generate stock-outs **(Figure 1).** The stochastic process resulting from this integration is then a rate-controlled process (non-homogeneous Markovian process. The mode of the machine i M can be described by a stochastic process ξ i (t), i =1,2 with value in B i = {1, 2}. Such a machine is available when it is operational (ξ i (t) = 1) and unavailable when it is under repair (ξ_i (t) = 2).

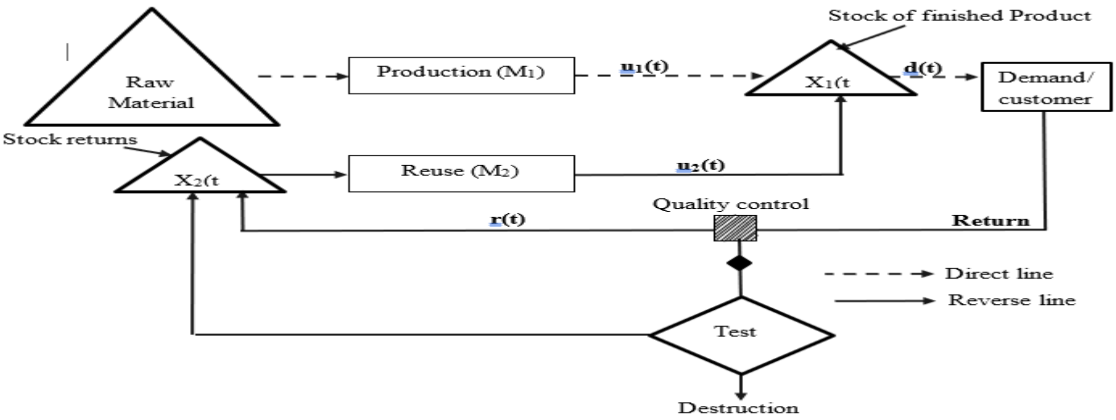

Fig 1. Structure of the production system

The transition diagrams, which describes the dynamics of the considered manufacturing system, are presented in Figure 2, 3 and 4. We then have $\xi(t) \in B = \{1, 2, 3, 4\}$. With $\lambda_{\alpha\beta}$ denoting a jump rate of the system from state α to state β, we can describe $\xi(t)$ statistically by the following state probabilities:

$$P[\xi(t+\delta t) = \beta \mid \xi(t) = \alpha] = \begin{cases} \lambda_{\alpha\beta}(.)\delta t + 0(\delta t) & if\ \alpha \neq \beta \\ 1 + \lambda_{\alpha\beta}(.)\delta t + 0(\delta t) & if\ \alpha \neq \beta \end{cases} \qquad (1)$$

where $\lambda_{\alpha\beta} \geq 0\ (\alpha \neq \beta), \lambda_{\alpha\alpha} = -\sum_{\alpha \neq \beta} \lambda_{\alpha\beta}$ and $\lim_{\delta t \to 0} \frac{0(\delta t)}{\delta t} = 0$ for alla $\alpha, \beta \in B$

Machine 1

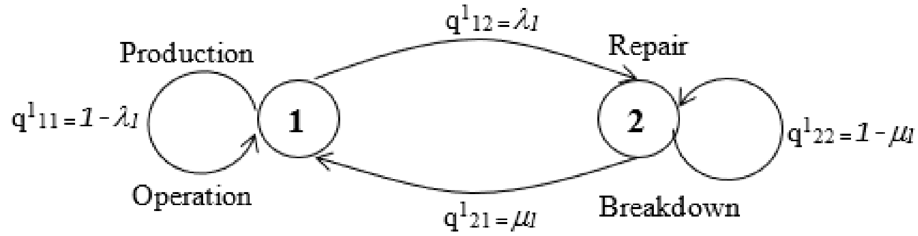

Fig 2. Machine State transition 1

Machine 2

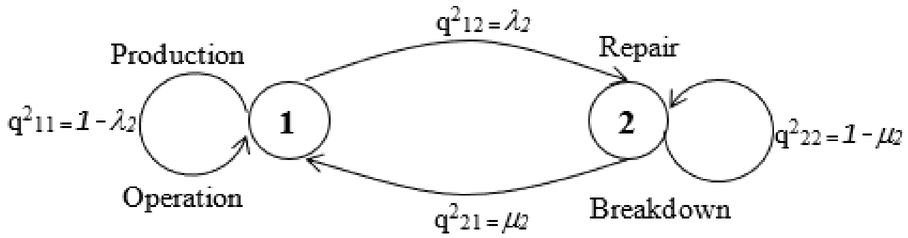

Fig 3. Machine State transition 1

With: $\mu_1 = \frac{1}{MTTR_1}$; $\lambda_1 = \frac{1}{MTTF_1}$; $\mu_2 = \frac{1}{MTTR_2}$; $\lambda_2 = \frac{1}{MTTF_2}$

By combining the two machines M1 and M2, we have a complete system:

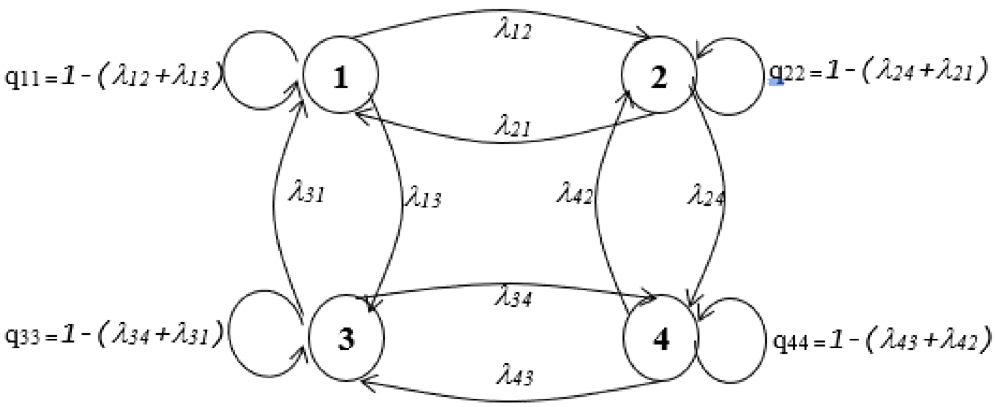

Fig 4. State transition of the system

With:

$q^1_{12} = \lambda_{13} = \lambda_{24}$ (failure rate of M_1) ; $q^1_{21} = \lambda_{31} = \lambda_{42}$ (corrective maintenance rate of M_1)

$q^2_{12} = \lambda_{12} = \lambda_{34}$ (failure rate of M_2) ; $q^2_{21} = \lambda_{21} = \lambda_{43}$ (corrective maintenance rate of M_2)

The operational mode of the manufacturing system can be described by the random vector $\xi(t) = \xi_1(t), \xi_2(t))$. Given that the dynamics of each machine is described by a 2-state stochastic process, the set of possible values of the process $\xi(t)$ can be determined from the values of $\xi_1(t)$ and $\xi_2(t)$, as illustrated in Table 2, with :

- Mode 1: M_1 and M_2 are operational
- Mode 2: M_1 is operational and M_2 is under repair
- Mode 3: M_1 is under repair and M_2 is operational
- Mode 4: M_1 and M_2 are under repair.

The dynamics of the system is described by a discrete element, namely $\xi(t)$ and a continuous element $x(t)$. The discrete element represents the status of the machines and the continuous one represents that of the stock level. It can be positive for an inventory or negative for a backlog.

Table 2 Modes of a two-machine manufacturing system

A. ξ_1(t)	**B. 1**	**C. 1**	**D. 2**	**E. 2**	F. Machine 1	G. Stochastic process
H. ξ_2(t)	**I. 1**	**J. 2**	**K. 1**	**L. 2**	**M.** Machine 2	**N.** Stochastic process
O. ξ(t)	**P. 1**	**Q. 2**	**R. 3**	**S. 4**	T. Manufacturing system	**U.** Stochastic process

We assume that the failure rate of M_1 depends on its productivity, and is defined by:

$$q^1_{12} = \begin{cases} \theta_1 \ if \ u_1 \in \beta[U, u_{1\max}] \\ \theta_2 \ if \ u_1 \in \beta[0, U] \end{cases} \quad with \ \theta_1 \geq \theta_2 \geq 0 \ and \ 0 \leq U \leq u_{1\max}$$

Hence, ξ(t) is described by the following matrix :

$$Q = \begin{cases} \Theta_1 \text{ if } u_1 \in \beta\,[U, u_{1\max}] \\ \Theta_2 \text{ if } u_1 \in \beta\,[0, U] \end{cases} \quad \text{with} \qquad (2)$$

$$\Theta_1 = \begin{pmatrix} -(q^2_{12} + \theta_1) & q^2_{12} & \theta_1 & 0 \\ q^2_{21} & -(q^2_{21} + \theta_1) & 0 & \theta_1 \\ q^1_{21} & 0 & -(q^1_{21} + q^2_{12}) & q^2_{12} \\ 0 & q^1_{21} & q^2_{21} & -(q^1_{21} + q^2_{21}) \end{pmatrix}$$

$$\Theta_2 = \begin{pmatrix} -(q^2_{12} + \theta_2) & q^2_{12} & \theta_2 & 0 \\ q^2_{21} & -(q^2_{21} + \theta_2) & 0 & \theta_2 \\ q^1_{21} & 0 & -(q^1_{21} + q^2_{12}) & q^2_{12} \\ 0 & q^1_{21} & q^2_{21} & -(q^1_{21} + q^2_{21}) \end{pmatrix}$$

With: $\theta_1 = \lambda_{13} = \lambda_{24}$; $\theta_2 = \lambda_{13} = \lambda_{24}$

The continuous part of the system dynamics is described by the following differential equation:

$$\frac{dx_1(t)}{dt} = u_1(t) + u_2(t) - d\,, \ \ x(0) = x_0 \qquad (3)$$

$$\frac{dx_2(t)}{dt} = r(t) - u_2(t) - disp \qquad avec \ \ x_2(0) = x_{20}; \ \ x_2(t) \geq 0 \qquad (4)$$

Where x_0 and d are the given initial stock level and demand rate, respectively.

Where x_{20}, r, *et* $disp$ are respectively the initial s tock of returned products, the return rate, and the rejection rate of returned products.

The set of the feasible control policies Γ, including $u_1(\cdot)$ and $u_2(\cdot)$, is given by:

$$\Gamma(\alpha) = \{u = (u_1, u_2) \in \mathbb{R}^2 / \ 0 \leq u_1(\cdot) \leq u_{1max} \ ; \ 0 \leq u_2(\cdot) \leq u_{2max}\} \qquad (5)$$

where $u_1(\cdot)$ and $u_2(\cdot)$ are known as control variables, and constitute the control policies of the problem under study. The maximal productivities of the main machine and the second machine are denoted by u_{1max}, and u_{2max} , respectively

Let g(.) be the cost rate defined as follows (equation 6) :

$$g(x(t), \alpha(t), u(t)) = c_1^+ x_1^+ + c_1^- x_1^- + c_2 x_2 \qquad (6)$$

$$with \quad x^+ = \max(0, x) \ ; \ x^- = \max(-x, 0)$$

The production planning problem considered in this paper involves the determination of the optimal control policies ($u_1^*(t)$ and $u_2^*(t)$) minimizing the expected discounted cost $J(\cdot)$ given by :

$$J(\alpha, x_1, x_2, u_1, u_2) = E\left\{\int_0^\infty e^{-\rho t} g(\alpha, x_1, x_2)dt \mid \alpha(0) = \alpha, x_1(0) = x_{10}, x_2(0) = x_{20}\right\} \quad (7)$$

$\forall\, u \in \Gamma(\alpha), \forall\, \alpha \in B$

where ρ is the discount rate. The value function of such a problem is defined as follows :

$$v(\alpha, x) = \inf_{u_1, u_2 \in \Gamma(\alpha)} J(\alpha, x, u_1, u_2) \quad \forall\, \alpha \in B \qquad (8)$$

The properties of the value function and the manner in which the Hamilton-Jacobi-Bellman (HJB) equations are obtained can be found in Kenné et al. [19], with a constant failure rate.

II. OPTIMALITY CONDITIONS

By successive developments, we arrive at the equation of HJB below like that presented in A. F. Koueudeu and J.P. Kenne [20] :

$$\rho v(\alpha, x_1, x_2) = \min_{u_1, u_2 \in \Gamma(\alpha)} \begin{bmatrix} g(\alpha, x_1, x_2) \sum_{\beta \neq B} \lambda_{\alpha\beta} v(x, \beta) + (u_1 + u_2 - d)\dfrac{\partial v(\alpha, x_1)}{\partial x_1} \\ +(r - u_2 - disp)\dfrac{\partial v(\alpha, x_2)}{\partial x_2} \end{bmatrix} \qquad (9)$$

In order to solve the problem numerically, the partial derived terms according to x_1 and x_2 are approximated according to Kushner's approach as follows:

$$\frac{\partial v(x_1, \alpha)}{\partial x_1} = \begin{cases} \dfrac{1}{h_1}\left(v^h(\alpha, x_1 + h_1, x_2) - v^h(\alpha, x_1, x_2)\right) & if\ (u_1 + u_2 - d) > 0 \\ \dfrac{1}{h_1}\left(v^h(\alpha, x_1, x_2) - v^h(\alpha, x_1 - h_1, x_2)\right) & otherwise \end{cases} \qquad (10)$$

$$\frac{\partial v(x_2, \alpha)}{\partial x_2} = \begin{cases} \dfrac{1}{h_2}\left(v^h(\alpha, x_1, x_2 + h_1) - v^h(\alpha, x_1, x_2)\right) & if\ (r - u_2 - disp) > 0 \\ \dfrac{1}{h_2}\left(v^h(\alpha, x_1, x_2) - v^h(\alpha, x_1, x_2 - h_1)\right) & otherwise \end{cases} \qquad (11)$$

Combining equations 10 and 11 in equation 9, we have the following equation 12:

$$v^h(\alpha, x_1, x_2) = \min_{(u_1, u_2) \in \Gamma^h(\alpha)} \left[\frac{\begin{matrix} g(\alpha, x_1, x_2) \sum_{\beta \neq B} \lambda_{\alpha\beta} v^h(x_1, x_2, \beta) + \\ \dfrac{(u_1 + u_2 - d)}{h_1}\begin{bmatrix} v^h(x_1 + h_1, x_2, \alpha) Ind\{u_1 + u_2 - d \geq 0\} \\ + v^h(x_1 - h_1, x_2, \alpha) Ind\{u_1 + u_2 - d < 0\} \end{bmatrix} \\ + \dfrac{(r - u_2 - disp)}{h_2}\begin{bmatrix} v^h(x_1, x_2 + h_1, \alpha) Ind\{r - u_2 - d \geq 0\} \\ + v^h(x_1, x_2 - h_1, \alpha) Ind\{r - u_2 - d < 0\} \end{bmatrix} \end{matrix}}{\rho + \dfrac{|u_1 + u_2 - d|}{h_1} + \dfrac{|r - u_2 - disp|}{h_2} + |\lambda_{\alpha\alpha}|} \right] \qquad (12)$$

The discretized equation (12) translates into four equations, (13, 14, 15 and 16) expressing the value functions of the production system composed of two machines subject to two states ($\alpha(t)$ = 1, 2, 3 et 4).

For the 4 modes, we will get:

- Mode 1

$$v^h(x_1,x_2,1) = \min_{u_1,u_2\in\Gamma(\alpha)} \left\{ \frac{\begin{array}{c} g(x_1,x_2,\alpha) + q_{13}v^h(x_1,x_2,1) + q_{12}v^h(x_1,x_2,2) \\ + \dfrac{[u_1+u_2-d]}{h_1}\begin{bmatrix} v^h(x_1+h_1,x_2,\alpha) * Ind\{u_1+u_2-d>0\} \\ +v^h(x_1-h_1,x_2,\alpha) * Ind\{u_1+u_2-d\le 0\} \end{bmatrix} \\ + \dfrac{[r-u_2-disp]}{h_2}\begin{bmatrix} v^h(x_1,x_2+h_2,\alpha) * Ind\{r-u_2-disp>0\} \\ +v^h(x_1,x_2-h_2,\alpha) * Ind\{r-u_2-disp\le 0\} \end{bmatrix} \end{array}}{\left(\rho + \dfrac{|u_1+u_2-d|}{h_1} + \dfrac{|r-u_2-disp|}{h_2} + |q_{11}|\right)} \right\} \tag{13}$$

- Mode 2

$$v^h(x_1,x_2,2) = \min_{u_1,u_2\in\Gamma(\alpha)} \left\{ \frac{\begin{array}{c} g(x_1,x_2,\alpha) + q_{24}v^h(x_1,x_2,0) + q_{21}v^h(x_1,x_2,3) \\ + \dfrac{[u_1+u_2-d]}{h_1}\begin{bmatrix} v^h(x_1+h_1,x_2,\alpha) * Ind\{u_1+u_2-d>0\} \\ +v^h(x_1-h_1,x_2,\alpha) * Ind\{u_1+u_2-d\le 0\} \end{bmatrix} \\ + \dfrac{[r-u_2-disp]}{h_2}\begin{bmatrix} v^h(x_1,x_2+h_2,\alpha) * Ind\{r-u_2-disp>0\} \\ +v^h(x_1,x_2-h_2,\alpha) * Ind\{r-u_2-disp\le 0\} \end{bmatrix} \end{array}}{\left(\rho + \dfrac{|u_1+u_2-d|}{h_1} + \dfrac{|r-u_2-disp|}{h_2} + |q_{22}|\right)} \right\} \tag{14}$$

- Mode 3

$$v^h(x_1,x_2,3) = \min_{u_1,u_2\in\Gamma(\alpha)} \left\{ \frac{\begin{array}{c} g(x_1,x_2,\alpha) + q_{34}v^h(x_1,x_2,0) + q_{31}v^h(x_1,x_2,3) \\ + \dfrac{[u_1+u_2-d]}{h_1}\begin{bmatrix} v^h(x_1+h_1,x_2,\alpha) * Ind\{u_1+u_2-d>0\} \\ +v^h(x_1-h_1,x_2,\alpha) * Ind\{u_1+u_2-d\le 0\} \end{bmatrix} \\ + \dfrac{[r-u_2-disp]}{h_2}\begin{bmatrix} v^h(x_1,x_2+h_2,\alpha) * Ind\{r-u_2-disp>0\} \\ +v^h(x_1,x_2-h_2,\alpha) * Ind\{r-u_2-disp\le 0\} \end{bmatrix} \end{array}}{\left(\rho + \dfrac{|u_1+u_2-d|}{h_1} + \dfrac{|r-u_2-disp|}{h_2} + |q_{33}|\right)} \right\} \tag{15}$$

- Mode 4

$$v^h(x_1,x_2,4) = \min_{u_1,u_2\in\Gamma(\alpha)} \left\{ \frac{\begin{array}{c} g(x_1,x_2,\alpha) + q_{43}v^h(x_1,x_2,1) + q_{31}v^h(x_1,x_2,2) \\ +\frac{[u_1+u_2-d]}{h_1}\begin{bmatrix} v^h(x_1+h_1,x_2,\alpha) * Ind\{u_1+u_2-d>0\} \\ +v^h(x_1-h_1,x_2,\alpha) * Ind\{u_1+u_2-d\le 0\}\end{bmatrix} \\ +\frac{[r-u_2-disp]}{h_2}\begin{bmatrix} v^h(x_1,x_2+h_2,\alpha) * Ind\{r-u_2-disp>0\} \\ +v^h(x_1,x_2-h_2,\alpha) * Ind\{r-u_2-disp\le 0\}\end{bmatrix}\end{array}}{\left(\rho+\frac{|u_1+u_2-d|}{h_1}+\frac{|r-u_2-disp|}{h_2}+|q_{44}|\right)} \right\} \quad (16)$$

For the parameters of the problem, we considered the following:

- The estate $D = \{x1 : -10 \le x1 \le 20 ; x2 : 0 \le x2 \le 20\}$
- Recovery rate: 50% of demand
- "Disposal" rate: 10% of recovered products
- The feasibility of the problem is tested according to the equation defined above.
- Table of numerical data used

Table 3 : Parameters of numerical example

C_1+	C_1-	C_2	hx_1	hx_2	U_{1max}	U_{2max}	d	λ_1	μ_1	λ_2	μ_2	ρ
10	50	2	0.5	0.5	1.3	1.15	1.25	1/60	1/15	1/60	1/15	0.1

Thus the $u(x_1,x_2,\alpha)$ production policies obtained indicate for the different configurations of stock states x 1 and *x* 2 and system states (1, *2* , 3 and 4), the production rates to be applied to the production machines M_1 and remanufacturing M_2. These policies are of the hedging point type. Below are the details of each of these policies.

- Mode 1

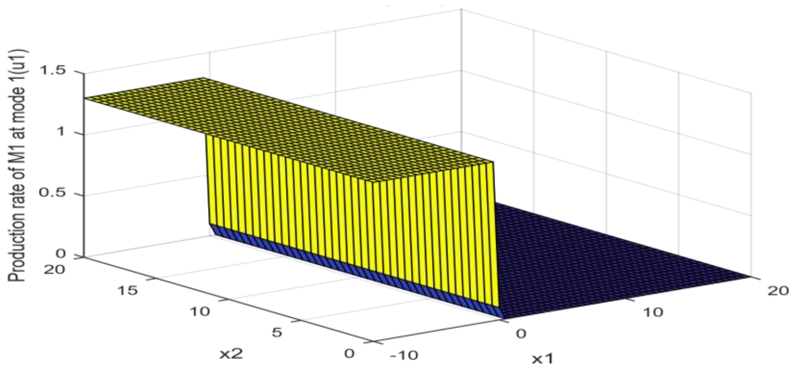

Fig 5. Production policy M_1 mode 1

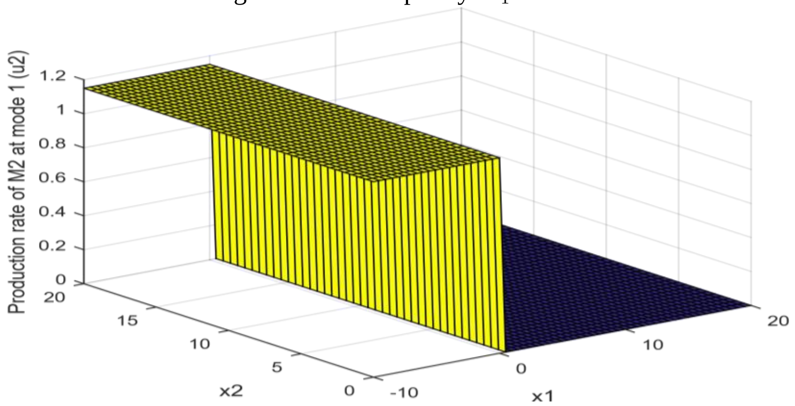

Fig 6. Production policy M_2 mode 1

In mode 1, the 2 machines M 1 and M_2 produce to meet demand. As illustrated above, the machine M 1 can produce in 3 thresholds according to the state of the stock x 1 defined by its critical thresholds Z_{13} and Z_{12}:

$$u_1(\alpha = 1, x_1, x_2) = \begin{cases} U_{1\max} & si\ x_1 < Z_{13} \\ d - U_{2\max} & si\ Z_{31} \leq x_1 \leq Z_{12} \\ 0 & si\ x_1 > Z_{32} \end{cases}$$

Whith $Z_{13} = 0\ et\ Z_{12} = 0{,}5$

The M_2 machine produces in addition to M_1 at its maximum rate or zero rate according to the policy below:

$$u_2(\alpha = 1, x_1, x_2) = \begin{cases} U_{2\max} & si\ x_1 \leq Z_{11} \\ \\ 0 & si\ x_1 > Z_{11} \end{cases}$$

With $Z_{11} = 0{,}5$

The value function is of the form below. The study presented will make it possible to evaluate sensitivity to variation of the main parameters.

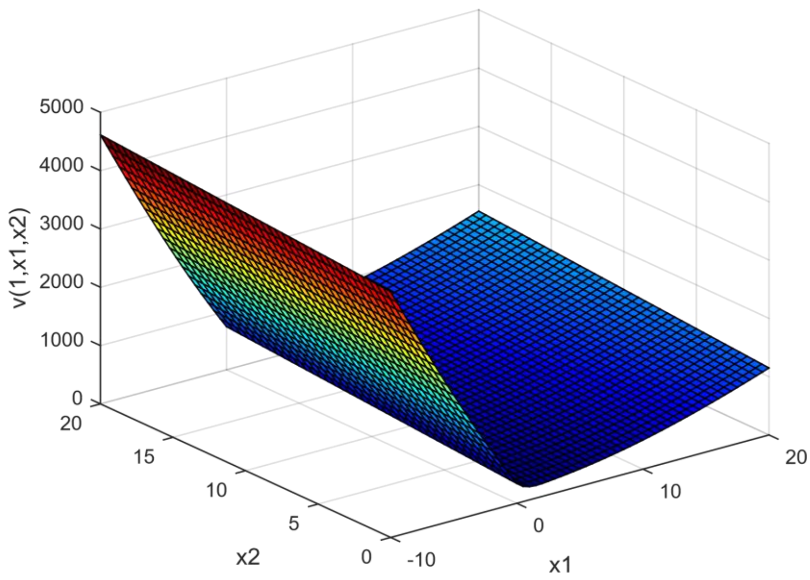

Fig 7. Value function at mode 1

- Mode 2

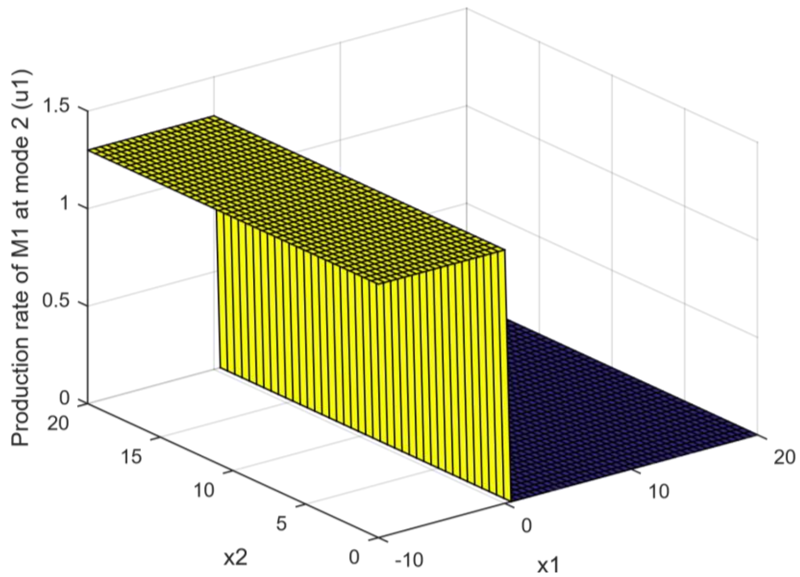

Fig 8. Production policy M_1 mode 2

In mode 2, the M1 machine is the only one to serve the demand, the M2 machine being in default. Its optimal policy is also critical threshold according to the diagram below:

$$u_1(\alpha = 2, x_1, x_2) = \begin{cases} U_{1\max} & si\ x_1 \le Z_2 \\ 0 & si\ x_1 > Z_2 \end{cases}$$

With $Z_2 = 0{,}5$

Below is the look of the value function in Mode 2.

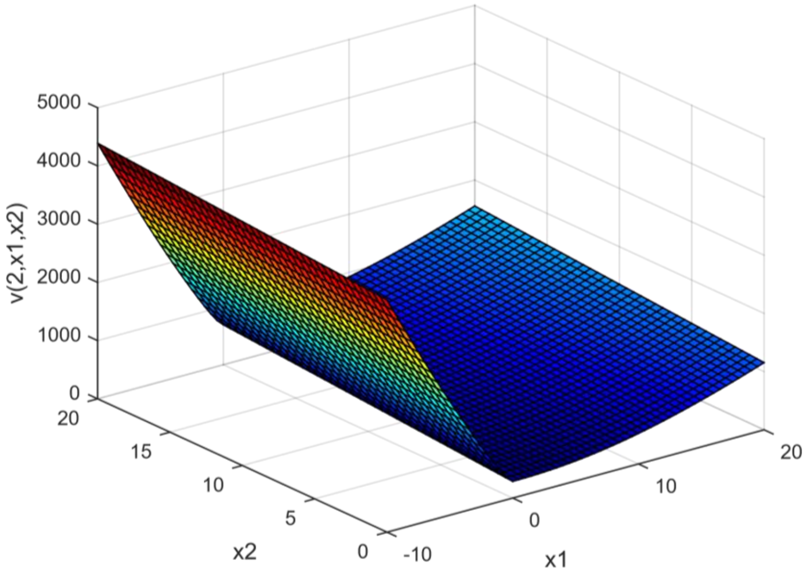

Fig 9. Value function at mode 2

- Mode 3

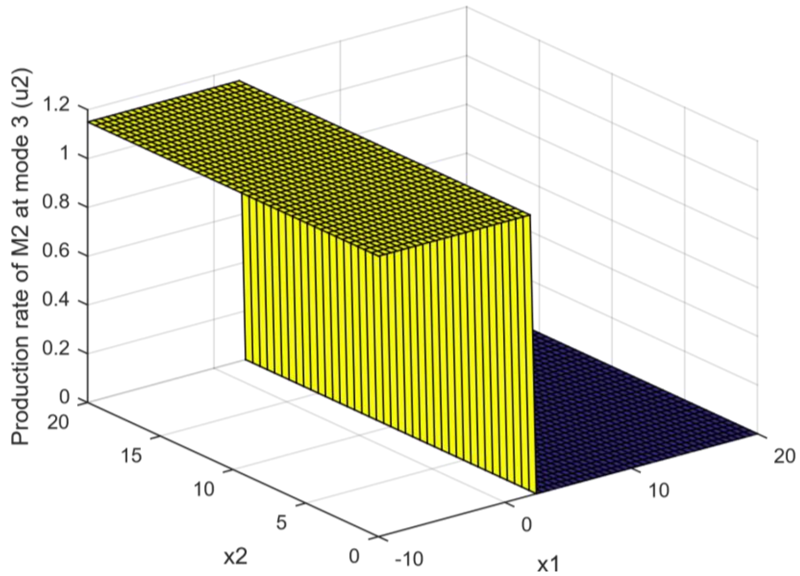

Fig 10. Production policy M_2 mode 3

In mode 1, the M2 machine is the only one to serve the demand, the M1 machine being in default. Its optimal policy is also critical threshold according to the diagram below :

$$u_2(\alpha = 2, x_1, x_2) = \begin{cases} U_{2\max} & si\ x_1 \le Z_2 \\ 0 & si\ x_1 > Z_2 \end{cases}$$

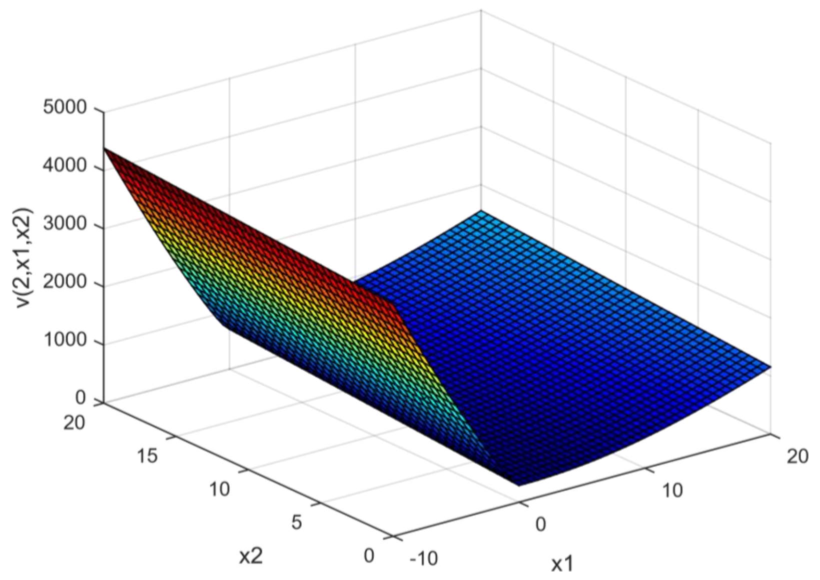

Fig 11. Value function at mode 3

With $Z_1 = 2{,}5$

The production rate of M2 being lower than that of M1, it is consistent that we case of defect of M1, we begin to produce more (Z1 = 2.5 against Z2 = 0.5 in mode 2).

Below is the look of the value function in Mode 3.

- Mode 4

In mode 4, the M1 and M2 machines are out of order, with zero production in their respective productions. The value function looks below.

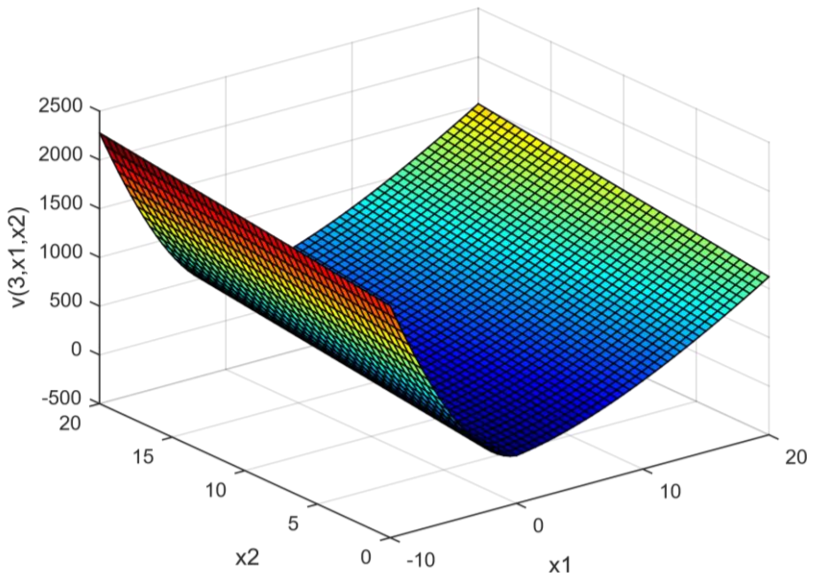

Fig 12. Value function at mode 4

Sensitivity analysis (inventory costs, shortage costs, etc.) As several parameters influence the model and therefore the optimal policies, it seems important to see how the latter would react in the event of a significant change in the latter. To do this, we performed a sensitivity analysis on the 3 parameters below. We recall that as it is rigorous for a sensitivity analysis, we varied only one parameter at a time and observed the evolution of the thresholds triggering the production stages, for each of the machines and each of the modes.

Cost of placing in inventory of the main stock

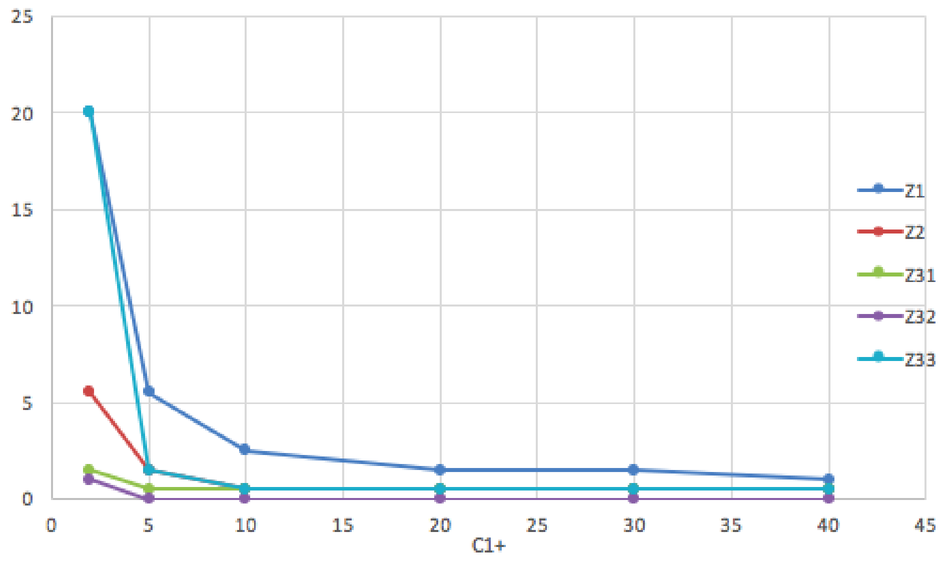

Fig 13. Sensibility C+

We notice that in general, the production tipping thresholds (corresponding to values of x1) all fall when we review upwards the cost of inventorying the main stock x1, which is completely consistent because with the increase in the cost of stocking, we want to limit as much as possible the quantities stored and therefore start producing as soon as possible. We also note that for a cost put in inventory less than 5 the machine M2 remains in permanent production whatever the values of stock x1 and x2. Only the Um1 production level of M1 is quite difficult to cross because M2 already satisfies demand in the majority of cases.

Main stock shortage cost

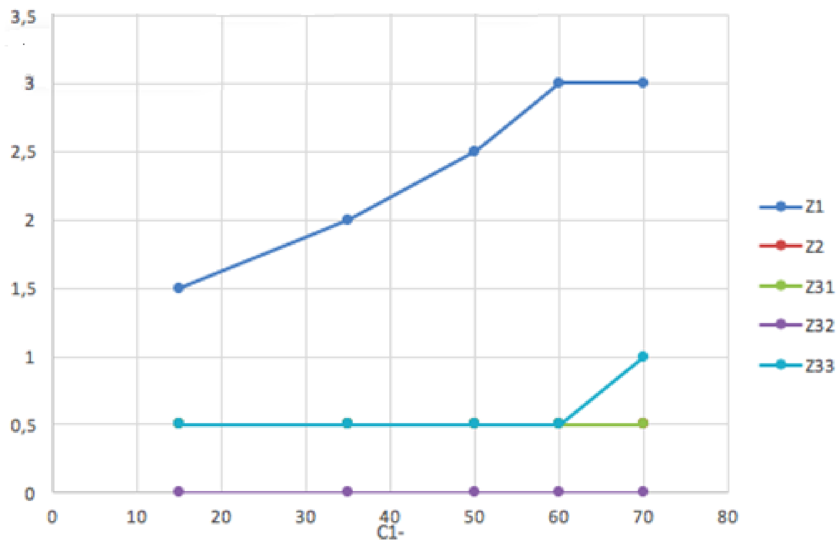

Fig 14. Sensibility C-

We note here that, as a general rule, the tipping thresholds of the different levels increase when the cost of stock-outs is increased, which is once again consistent because the model will want to limit as much as possible the additional costs related to the shortage. This is even more noticeable in mode 1 where is Z1 is the threshold for putting the M2 machine into production.

Recovery rate (Recycling)

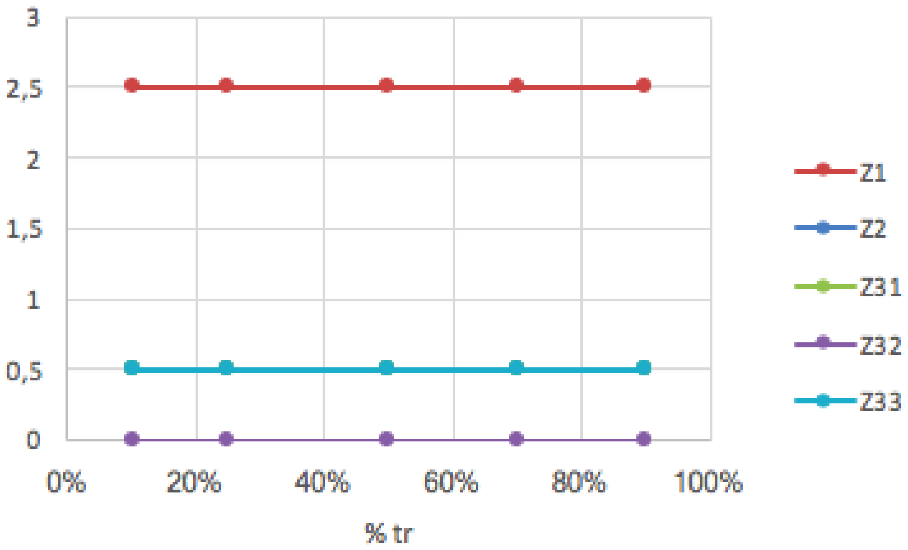

Fig 15. Sensitivity on recycling rate

We observe here that the tipping thresholds of the different levels of machine production are completely insensitive to variations in the recycling rate in this area.

III. DISCUSSION ON POSSIBLE EXTENSIONS

At the end of this study aimed at understanding the behavior of this system of 2 machines integrating reverse logistics, it seems interesting to ask what other aspects could be addressed downstream of this work or be integrated into it to make this model even more robust.Indeed, this study gives us very interesting results on the optimal management approach of our system for an optimized satisfaction of demand and effective cost control. The next step could be to move to the experimental approach, for simulation of optimal policies obtained from the mathematical model and validation of effectiveness in the field. This step effectively makes it possible to answer the "what if?" In other words, what response will we have from the system if the optimal policy suggested by the mathematical model is applied. This would be a kind of validation step. It may be that at the end of this experiment, some adjustments must be made to the mathematical solution before implementation in the field.

The other important aspect that we could have integrated is the consideration of the maintenance policy. Indeed, our model assumes that the failure and repair rates of machines are constant, which is not always the case, they can vary according to several criteria such as the rate of production, the age of the machine and many others. Thus, a model incorporating these considerations, although more complex, would be even more representative of the reality on the ground. Similarly, since our study was established over an infinite horizon, we could finally have assumed a context where the maximum production rates of machines decreased increasing with the age of machines (which is a reality in some industrial contexts), and thus created two additional states corresponding to the replacement of machines when a maximum production rate falls below a certain threshold. This hypothesis would therefore add 2 additional states to our model and would make it more appropriate to this type of context.

IV. CONCLUSION

Current models for optimizing manufacturing systems that integrate the return of used products into their production system have some shortcomings. Regular use of production units at full capacity has an impact on the availability and reliability of the manufacturing system. In order to remedy these shortcomings, this article aimed to propose pragmatic models to solve the problems of optimization of production/reuse systems in a stochastic dynamic context.In this paper, our work has made a significant scientific contribution by reformulating existing mathematical models to incorporate the failure rate dependent on the production rate in the context of hybrid production/reuse systems subject to random breakdowns and repairs. The results of our work have been confirmed through studies by modeling, numerical resolution and sensitivity analysis on cases of flexible manufacturing systems.

This work confirmed that by integrating degradation based on the number of failures or productivity into a manufacturing system; and by controlling maintenance operations, in addition to the reliability of the system that is assured, the system becomes less vulnerable to changes in the costs of shortages, inventory and maintenance by continuously meeting demand. Our work was concluded by validating the policies proposed to manufacturers of printer ink cartridges. These contributions provide a solid basis for future work.

.

REFERENCES

[1] Kiesmüller, G. P. 2003. « A new approach for controlling a hybrid stochastic manufacturing/remanufacturing system with inventories and different leadtimes ». *European Journal of Operational Research*, vol. 147, no 1, p. 62-71.

[2] Lambert and Riopel, D. 2003. «Logistique inverse: revue de littérature ». *Coll. Les cahiers du GERAD, G-2003-61*. Montréal: École Polytechnique de Montréal, 45 p.

[3] Stock, J.R. 1992. «Reverse Logistics». *Council of Logistics Management*, Oak Brook, IL.

[4] Fleischmann. 2001. « The Impact of Product Recovery on Logistics Network Design ». *Production and Operations Management*, vol. 10, no 2, p. 156-173.

[5] Bennekrouf, L. Benyoucef and Sari, Z. 2010. «Problèmes de conception et pilotage des chaînes logistiques inverses et globales:Etat de l'art». I*n 8e Conférence Internationale de MOdélisation et SIMulation - (MOSIM'10)*. (Hammamet - Tunisie, Mai. 10-12 2010) : Évaluation et optimisation des systèmes innovants de production de biens et de services..

[6] Carter, C. and Ellram. 1998. « Reverse logistics: A review of literature and framework for future investigation ». *Journal of of Business Logisticsy*, vol. 19, no 1, p. 85-102.

[7] Abdessalem, D. Riopel and Hadj-Alouane, A. 2007. «Conception pour la logistique inverse : proposition d'un cahier des charges». (*CIRRELT-2007-26)*. Montréal: Centre interuniversitaire de recherche sur les réseaux d'entreprise, la logistique et le transport.

[8] Hajji, A. 2007. «Stratégies de production manufacturière dans un environnement de chaîne d'approvisionnement: Approche dynamique stochastique». Thèse de Doctorat en génie de la production automatisée, Montréal, École de Technologie Superieure, 234p.

[9] Min, H. and Gengui, Z. 2002. « Supply chain modeling: Past, present and future ». *Computers and Industrial Engineering*, vol. 43, p. 231-249.

[10] Mahadevan, B., David, F. P. and Moritz, F. 2003. « Periodic review, push inventory policies for remanufacturing ». *European Journal of Operational Research*, vol. 151, no 3, p. 536-551.

[11] Salema, M. I., Gomes, A. P. B-P. and Augusto, Q. N. 2007. « An optimization model for the design of a capacitated multi-product reverse logistics network with uncertainty ». *European Journal of Operational Research*, vol. 179, no 3, p. 1063-1077.

[12] Van der Laan, E. and Marc, S. 1997. « Production planning and inventory control with remanufacturing and disposal ». *European Journal of Operational Research*, vol. 102, no 2, p. 264-278.

[13] Gershwin, S. B. 1994. «Manufacturing systems engineering». *Englewood Cliffs, NJ*: prentice-Hall, New Jersey.

[14] Gharbi, A., R. Pellerin and Sadr, J. 2008. « Production rate control for stochastic remanufacturing systems ». *International Journal of Production Economics*, vol. 112, no 1, p. 37-47.

[15] Berthaut, F., R. Pellerin and Gharbi, A. 2009. «Control of a repair and overhaul system with probabilistic parts availability». *Production Planning and Control*, vol. 20, no 1, p. 57-67.

[16] Pellerin, R., Sadr, J., Gharbi, A. and Malhamé, R. 2009. « A production rate control policy for stochastic repair and remanufacturing systems ». *International Journal of Production Economics*, vol. 121, no 1, p. 39-48.

[17] Tongzhu, Z., W. Xueping, C. Jiangwei and Pengfei, C. 2010. « Remanufacturing mode and its reliability for the design of automotive products ». *5th International Conference on Responsive Manufacturing - Green Manufacturing (ICRM 2010)*. (Piscataway, NJ, USA, Jan. 11-13 2010) : IET.

[18] Kenné, J.-P., Dejax, P. and Gharbi, A. 2012. « Production planning of a hybrid manufacturing-remanufacturing system under uncertainty within a closed-loop supply chain ». *International Journal of Production Economics*, vol. 135, no 1, p. 81-93.

[19] Kenne, J. P., Boukas, E. K. and Gharbi, A. 2003. « Control of production and corrective maintenance rates in a multiple-machine, multiple-product manufacturing system ». *Mathematical and Computer Modelling*, vol. 38, p. 351-365.

[20] Kouedeu, A. F., Kenne, J-P., Dejax, P., Songmene, V. and Polotski, V. 2013a. « Analytical modelling and control of a hybrid manufacturing/remanufacturing system ». *7th IFAC Conference on Manufacturing Modelling, Management, and Control, (MIM 2013 - Proceedings)*, p.401-406.