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INTERNAL CUBIC PICTURE FUZZY SOFT MATRICES

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ABSTRACT: In this manuscript, we scrutinize the P-ordered and R-ordered internal cubic picture fuzzy soft matrices(ICPFSMs), which is a sequel expanded concept of cubicPicture Fuzzy soft matrix (CPFSM). Furthermore, some desirable properties on P-order and R-order ofICPFSM are investigated.

KEYWORDS: Cubic Soft Matrices, Internal Cubic Soft Set, External Cubic Soft Set, Picture Fuzzy Set, Picture Fuzzy Matrix, Cubic Picture Fuzzy Soft Matrices.2010 AMS Classification: 03E72,62C86.

I. INTRODUCTION

Zadeh[11] introduced the theory of Fuzzy set, consequently he studied the concept of interval-valued Fuzzy sets to capture the uncertainty of membership values. Atanossov's[1] proposed the concept of intuitionistic fuzzy sets(IFS) can deal the incomplete information of both the truth membership and non-membership values respectively. In 1999, Molodtsov[10] approaches the theory of Soft Set(SS) which has a rich potential formodelling uncertainty and vagueness. Maji et al.,[9] introduced the concept of fuzzy soft set(FSS).Picturefuzzy set(PFS) was introduced by Cuong[2], which is appropriate for such a situation.Dogra et.al.,[7] proposed to represent the Picture fuzzy matrix(PFM), which play an important role for new research in mathematical science and technology.June et.al.,[8] studied the concept of Cubic Sets(CSs) which was very appropriate for such a situation. Cubic soft matrix(CSM) was utterly new concept that was coined by Chinnadurai and Barkavi[3,4,5], they also extended this concept into internal and external cubic soft matrix and some of its theoretical properties are investigated by them. In recent years, Chinnaduraiand Madhanraj[6],defined cubic picture fuzzy soft set(CPFSS) and cubic picture fuzzy soft matrices(CPFSMs).In this manuscript our intention is to define theP-order and R-order of internal cubicpicture fuzzy soft Matrices. Furthermore, we scrutinized some desirable properties of ICPFSM.

II. PRELIMINARIES

Definition 2.1.[7]Let U be a non – empty set, E be a set of Parameters, A cubic Soft set over U is defined as a Pair (F, A) where F: A $\rightarrow$ P (U) and A $\subseteq$ E, then the Pair (F, A) can be represented as, (F, A) = {F(e) / e $\in$ A}, where F(e) = {< u, $\tilde{I}_e$ (u), I_e (u) > / u $\in$ U, e $\in$ A } is as interval valued fuzzy set and $\tilde{I}_e$(u) is a fuzzy set.

Definition 2.2. [7] Let U be the Universal set and E be the a set of parameters. A Cubic soft set (F, A) = F(e) = $\{< u, \underline{I_e}(u), \overline{I_c}(u) >/u \in U, e \in A\}$ over U is set to be an Interval Cubic Soft set if $\underline{I_e}(u) \leq I_e(u) \leq \overline{I_c}(u)$for all e $\in$ A and for all u $\in$U.

III. INTERNAL CUBIC PICTURE FUZZY SOFT SETS AND INTERNAL CUBIC PICTURE FUZZY SOFT MATRICES

In this section, we define an Internal cubic picture-fuzzy soft setsand Internal cubic picture-fuzzy soft matrices and its order relations are discussed.

Definition 3.1. Let U be a non empty set, E be a set of parameters and A $\subseteq$ E. An Internal cubic Picture Fuzzy Soft Set over U is defined as a pair (F, A), where F: A $\rightarrow$ P^u, (F, A) = {F(e) / e $\in$ A}, where F (e) = $\Big\{< \left[\underline{I}^{\alpha}_{F(e)}, \bar{I}^{\alpha}_{F(e)}\right] I^{\alpha}_{F(e)}, >, < \left[\underline{I}^{\beta}_{F(e)}, \bar{I}^{\beta}_{F(e)}\right] I^{\beta}_{F(e)}, >, < \left[\underline{I}^{\gamma}_{F(e)}, \bar{I}^{\gamma}_{F(e)}\right] I^{\gamma}_{F(e)}, >\Big\}$, Also satisfying the condition $\underline{I}^{\alpha}_{F(e)} \leq I^{\alpha}_{F(e)} \leq \bar{I}^{\alpha}_{F(e)}, \underline{I}^{\beta}_{F(e)} \leq I^{\beta}_{F(e)} \leq \bar{I}^{\beta}_{F(e)}, \underline{I}^{\gamma}_{F(e)} \leq I^{\gamma}_{F(e)} \leq \bar{I}^{\gamma}_{F(e)}$.

Definition 3.2. The complement of an ICPFSS (F, A) denoted by (F, A)^c is defined as,

$$(\mathrm{F,A})^C = \{< x, \left[\underline{I}^{\gamma}_{F(e)}(x), \bar{I}^{\gamma}_{F(e)}(x)\right] I^{\gamma}_{F(e)}(x), >, < \left[\underline{I}^{\beta}_{F(e)}(x), \bar{I}^{\beta}_{F(e)}(x)\right] I^{\beta}_{F(e)}(x) >,$$
$$< \left[\underline{I}^{\alpha}_{F(e)}(x), \bar{I}^{\alpha}_{F(e)}(x)\right] I^{\alpha}_{F(e)}(x) > \forall x \in U and e \in A\}$$

Definition 3.3. Let U = {u_1, u_2…., u_n}Universal set and E = {e_1, e_2…., e_n} be a set of parameters and A⊆ E, then the internal cubic picture fuzzysoft matrix (F, A) is represented in matrix form as,

$$I_C^P = [P_{ij}^I] = \begin{bmatrix} P_{11}^I & P_{12}^I & \dots & P_{1n}^I \\ P_{21}^I & P_{22}^I & \dots & P_{2n}^I \\ . & & & \\ . & & & \\ . & & & \\ P_{m1}^I & P_{m2}^I & \dots & P_{mn}^I \end{bmatrix}$$

Where, $I_C^P = [P_{ij}^I] = (< [\tilde{I}_{\alpha ij}^P, I_{\alpha ij}^P], [\tilde{I}_{\beta ij}^P, I_{\beta ij}^P], [\tilde{I}_{\gamma ij}^P, I_{\gamma ij}^P] >)$
$= \left(< \left[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P\right], I_{\alpha ij}^P >, < \left[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P\right], I_{\beta ij}^P >, < \left[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P\right], I_{\gamma ij}^P >\right)$ since i = 1, 2, …., m, j = 1, 2, …, n.

Where, $\underline{I}_{\alpha ij}^P \le \underline{I}_{\alpha ij}^P \le I_{\alpha ij}^P, \underline{I}_{\beta ij}^P \le \underline{I}_{\varphi ij}^P \le I_{\beta ij}^P, \underline{I}_{\gamma ij}^P \le \underline{I}_{\gamma ij}^P \le I_{\gamma ij}^P$ *and* also satisfies the condition, $0 < \overline{I}_{\alpha ij}^P + \overline{I}_{\beta ij}^P + \overline{I}_{\gamma ij}^P \le 1$, then I_C^P is an (mxn) ICPFSM

Definition 3.3. Consider the ICPFSM

$P_C^P = \left\{\left[< \left[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P\right], I_{\alpha ij}^P >, < \left[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P\right], I_{\beta ij}^P >, < \left[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P\right], I_{\gamma ij}^P >\right\}_{(m\times n)}$, then the complement of the ICPFSM is, $(I_C^P)^C = \left\{< \left[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P\right], I_{\gamma ij}^P >, < \left[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P\right], I_{\beta ij}^P >, < \left[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P\right], I_{\alpha ij}^P >\right\}_{(m\times n)}$

Example 3.4.

$$I_C^P = \begin{bmatrix} < [0.20,0.30],0.25, [0.10,0.40],0.37, [0.12,0.30],0.16 > \\ < [0.30,0.40],0.32, [0.11,0.30],0.23, [0.15,0.30],0.29 > \\ < [0.20,0.25],0.24, [0.38,0.40],0.39, [0.19,0.35],0.32 > \end{bmatrix}$$

$$\begin{bmatrix} < [0.18,0.40],0.36, [0.21,0.34],0.28, [0.14,0.26],0.25 > \\ < [0.21,0.36],0.30, [0.13,0.31],0.30, [0.18,0.28],0.19 > \\ < [0.10,0.19],0.17, [0.22,0.32],0.25, [0.19,0.33],0.27 > \end{bmatrix}$$

$$(I_C^P)^C = \begin{bmatrix} < [0.12,0.30],0.16, [0.10,0.40],0.37, [0.20,0.30],0.25 > \\ < [0.15,0.30],0.29, [0.11,0.30],0.23, [0.30,0.40],0.32 > \\ < [0.19,0.35],0.32, [0.38,0.40],0.39, [0.20,0.25],0.24 > \end{bmatrix}$$

$$\begin{bmatrix} < [0.14,0.26], 0.25, [0.21,0.34], 0.28, [0.18,0.40] > \\ < [0.18,0.28],0.19, [0.13,0.31],0.30, [0.21,0.36],0.30 > \\ < [0.19,0.33], 0.27, [0.22,0.32], 0.25, [0.10,0.19],0.17 > \end{bmatrix}$$

IV. P-order and R-order of an ICPFSM

In this section, P-order and R-order of an internal cubic picture Fuzzy soft matrices are defined and their related properties are investigated.

Definition 4.1. *Let* $I_C^P = \left\{\left[< \left[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P\right], I_{\alpha ij}^P >, < \left[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P\right], I_{\beta ij}^P >, < \left[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P\right], I_{\gamma ij}^P >\right\}$ and $I_C^Q = \left\{\left[< \left[\underline{I}_{\mu ij}^Q, \overline{I}_{\mu ij}^Q\right], I_{\mu ij}^Q >, < \left[\underline{I}_{\eta ij}^Q, \overline{I}_{\eta ij}^Q\right], I_{\eta ij}^Q >, < \left[\underline{I}_{\xi ij}^Q, \overline{I}_{\xi ij}^Q\right], I_{\xi ij}^Q >\right\}$ *all* $\in ICPFSM_{(m\times n)}$, then1.P- union of I_C^P and I_C^Q is defined by $I_C^P \vee_P I_C^Q = I_C^M$, where

$I_C^M = \left[max\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, \right\}, max\left\{< \overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P >, < \overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q >, \right\} max\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, \right\}\right.$ for all *i, j*.

2. P- intersection of I_C^P and I_C^Q is defined by $I_C^P \wedge_P I_C^Q = I_C^M$, where$I_C^M = \Big[min\Big\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, \Big\}, min\Big\{< \overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P >, < \overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q >, \Big\} min\Big\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, \Big\}$ for all *i*, *j*.

3. R- union of I_C^P and I_C^Q is defined by $I_C^P \vee_R I_C^Q = I_C^M$, where$I_C^M = \Big[max\Big\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, \Big\}, max\Big\{< \overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P >, < \overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q >, \Big\} min\Big\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, \Big\}$ for all *i*, *j*.

4. R- intersection of I_C^P and I_C^Q is defined by $I_C^P \wedge_R I_C^Q = I_C^M$, where$I_C^M = \Big[min\Big\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, \Big\}, min\Big\{< \overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P >, < \overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q >, \Big\} max\Big\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, \Big\}$ for all *i*, *j*.

Definition 4.4.

$Let\ I_C^P = (< [\tilde{I}_{\alpha ij}^P, I_{\alpha ij}^P], [\tilde{I}_{\beta ij}^P, I_{\beta ij}^P], [\tilde{I}_{\gamma ij}^P, I_{\gamma ij}^P] >)\ and$

$I_C^Q = \Big(< [\tilde{I}_{\mu ij}^Q, I_{\mu ij}^Q], [\tilde{I}_{\eta ij}^Q, I_{\eta ij}^Q], \Big[\tilde{I}_{\xi ij}^Q, I_{\xi ij}^Q\Big] >\Big) \in ICPFSM_{(m\times n)}$

$$then I_C^P and I_C^Q aresaidtobeP - orderICPFSM, denotedby I_C^P \subseteq_P I_C^Q \Leftrightarrow \overline{I}_{\alpha ij}^P \le \overline{I}_{\mu ij}^Q, \overline{I}_{\beta ij}^P \le \overline{I}_{\eta ij}^Q,$$
$$\overline{I}_{\gamma ij}^P \le \overline{I}_{\xi ij}^Q\ and\ I_{\alpha ij}^P \le I_{\mu ij}^Q, I_{\beta ij}^P \le I_{\eta ij}^Q,$$
$$I_{\gamma ij}^P \le I_{\xi ij}^P\ then\ the\ class\ of\ all\ p - ordered\ (ICPFSM)_P$$

Definition 4.5

$Let\ I_C^P = (< [\tilde{I}_{\alpha ij}^P, I_{\alpha ij}^P], [\tilde{I}_{\beta ij}^P, I_{\beta ij}^P], [\tilde{I}_{\gamma ij}^P, I_{\gamma ij}^P] >)\ and$

$I_C^Q = \Big(< [\tilde{I}_{\mu ij}^Q, I_{\mu ij}^Q], [\tilde{I}_{\eta ij}^Q, I_{\eta ij}^Q], \Big[\tilde{I}_{\xi ij}^Q, I_{\xi ij}^Q\Big] >\Big) \in ICPFSM_{(m\times n)}$

$$then I_C^P and I_C^Q aresaidtobeP - orderICPFSM, denotedby I_C^P \subseteq_P I_C^Q \Leftrightarrow \overline{I}_{\alpha ij}^P \le \overline{I}_{\mu ij}^Q, \overline{I}_{\beta ij}^P \le \overline{I}_{\eta ij}^Q,$$
$$\overline{I}_{\gamma ij}^P \le \overline{I}_{\xi ij}^Q\ and\ I_{\alpha ij}^P \ge I_{\mu ij}^Q, I_{\beta ij}^P \ge I_{\eta ij}^Q,$$
$$I_{\gamma ij}^P \ge I_{\xi ij}^P\ then\ the\ class\ of\ all\ p - ordered\ (ICPFSM)_R$$

V.SOME PROPERTIES ON ICPFSM

In this section,we discuss the P- order and R-order of Union and Intersection of the internalcubic picture fuzzy soft matrices and their related theoretical operations are investigated.

$\boldsymbol{Property}\ \mathbf{5.1}\ \ Let\ I_C^P, I_C^Q, I_C^R \in ICPFSM_{(m\times n),} then$

$(i)\ I_C^P \vee_P I_C^Q \vee_P I_C^R \in ICPFSM_{(m\times n),}$

$(ii)\ I_C^P \wedge_P I_C^Q \wedge_P I_C^R \in ICPFSM_{(m\times n),}$

$I_C^P = \Big\{\Big[< \Big[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P\Big], I_{\alpha ij}^P >, < \Big[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P\Big], I_{\beta ij}^P >, < \Big[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P\Big], I_{\gamma ij}^P >\Big\}$

Proof:

Here, $\underline{I}^P_{\alpha ij} \le I^P_{\alpha ij} \le \bar{I}^P_{\alpha ij},\ \underline{I}^P_{\beta ij} \le I^P_{\beta ij} \le \bar{I}^P_{\beta ij},\ \underline{I}^P_{\gamma ij} \le I^P_{\gamma ij} \le \bar{I}^P_{\gamma ij}$

$I^Q_C = \left\{\left[< \left[\underline{I}^Q_{\mu ij}, \bar{I}^Q_{\mu ij}\right], I^Q_{\mu ij} >< \left[\underline{I}^Q_{\eta ij}, \bar{I}^Q_{\eta ij}\right], I^Q_{\eta ij} >< \left[\underline{I}^Q_{\xi ij}, \bar{I}^Q_{\xi ij}\right], I^Q_{\xi ij} >\right\}\right.$ Here, $\underline{I}^Q_{\mu ij} \le I^Q_{\mu ij} \le \bar{I}^Q_{\mu ij},\ \underline{I}^Q_{\eta ij} \le I^Q_{\eta ij} \le \bar{I}^Q_{\eta ij}$,

$\underline{I}^Q_{\gamma ij} \le I^Q_{\gamma ij} \le \bar{I}^Q_{\gamma ij}$

$I^R_C = \left\{\left[< \left[\underline{I}^R_{\rho ij}, \bar{I}^R_{\rho ij}\right], I^R_{\rho ij} >< \left[\underline{I}^R_{\sigma ij}, \bar{I}^R_{\sigma ij}\right], I^R_{\sigma ij} >< \left[\underline{I}^R_{\tau ij}, \bar{I}^R_{\tau ij}\right], I^R_{\tau ij} >\right\}\right.$

$Here, \underline{I}^R_{\rho ij} \le I^R_{\rho ij} \le \bar{I}^R_{\rho ij}, \underline{I}^R_{\sigma ij} \le I^R_{\sigma ij} \le \bar{I}^R_{\sigma ij}, \underline{I}^R_{\tau ij} \le I^R_{\tau ij} \le \bar{I}^R_{\tau ij}\ all \in$ ICPFSM(m x n)

$(i) \Rightarrow max\left\{< \underline{I}^P_{\alpha ij}, \underline{I}^P_{\beta ij}, \underline{I}^P_{\gamma ij} >, < \underline{I}^Q_{\mu ij}, \underline{I}^Q_{\eta ij}, \underline{I}^Q_{\xi ij} >, < \underline{I}^R_{\rho ij}, \underline{I}^R_{\sigma ij}, \underline{I}^R_{\tau ij} >\right\}$

$\le max\left\{< I^P_{\alpha ij}, I^P_{\beta ij}, I^P_{\gamma ij} >, < I^Q_{\mu ij}, I^Q_{\eta ij}, I^Q_{\xi ij} >, < I^R_{\rho ij}, I^R_{\sigma ij}, I^R_{\tau ij} >\right\}$

$\le max\left\{< \bar{I}^P_{\alpha ij}, \bar{I}^P_{\beta ij}, \bar{I}^P_{\gamma ij} >, < \bar{I}^Q_{\mu ij}, \bar{I}^Q_{\eta ij}, \bar{I}^Q_{\xi ij} >, < \bar{I}^R_{\rho ij}, \bar{I}^R_{\sigma ij}, \bar{I}^R_{\tau ij} >\right\}$

Now, we have, $I^P_C \vee_P I^Q_C \vee_P I^R_C = I^M_C \Rightarrow max\left\{< \tilde{I}^P_{\alpha ij}, \tilde{I}^P_{\beta ij}, \tilde{I}^P_{\gamma ij} >, < \tilde{I}^Q_{\mu ij}, \tilde{I}^Q_{\eta ij}, \tilde{I}^Q_{\xi ij} >, < \tilde{I}^R_{\rho ij}, \tilde{I}^R_{\sigma ij}, \tilde{I}^R_{\tau ij} >\right\}$

$$\le max\left\{< I^P_{\alpha ij}, I^P_{\beta ij}, I^P_{\gamma ij} >, < I^Q_{\mu ij}, I^Q_{\eta ij}, I^Q_{\xi ij} >, < I^R_{\rho ij}, I^R_{\sigma ij}, I^R_{\tau ij} >\right\}$$

$Hence\ I^P_C \vee_P I^Q_C \vee_P I^R_C \in$ ICPFSM$_{(m \times n)}$

(ii) Let $I^P_C, I^Q_C and I^R_C \in$ ICPFSM$_{(m \times n)}$

$$\Rightarrow min\left\{< \underline{I}^P_{\alpha ij}, \underline{I}^P_{\beta ij}, \underline{I}^P_{\gamma ij} >, < \underline{I}^Q_{\mu ij}, \underline{I}^Q_{\eta ij}, \underline{I}^Q_{\xi ij} >, < \underline{I}^R_{\rho ij}, \underline{I}^R_{\sigma ij}, \underline{I}^R_{\tau ij} >\right\}$$
$$\le min\left\{< I^P_{\alpha ij}, I^P_{\beta ij}, I^P_{\gamma ij} >, < I^Q_{\mu ij}, I^Q_{\eta ij}, I^Q_{\xi ij} >, < I^R_{\rho ij}, I^R_{\sigma ij}, I^R_{\tau ij} >\right\}$$
$$\le min\left\{< \bar{I}^P_{\alpha ij}, \bar{I}^P_{\beta ij}, \bar{I}^P_{\gamma ij} >, < \bar{I}^Q_{\mu ij}, \bar{I}^Q_{\eta ij}, \bar{I}^Q_{\xi ij} >, < \bar{I}^R_{\rho ij}, \bar{I}^R_{\sigma ij}, \bar{I}^R_{\tau ij} >\right\}$$

Now, we have, $I^P_C \wedge_P I^Q_C \wedge_P I^R_C = I^M_C$

$\Rightarrow min\left\{< \tilde{I}^P_{\alpha ij}, \tilde{I}^P_{\beta ij}, \tilde{I}^P_{\gamma ij} >, < \tilde{I}^Q_{\mu ij}, \tilde{I}^Q_{\eta ij}, \tilde{I}^Q_{\xi ij} >, < \tilde{I}^R_{\rho ij}, \tilde{I}^R_{\sigma ij}, \tilde{I}^R_{\tau ij} >\right\}$

$$\le min\left\{< I^P_{\alpha ij}, I^P_{\beta ij}, I^P_{\gamma ij} >, < I^Q_{\mu ij}, I^Q_{\eta ij}, I^Q_{\xi ij} >, < I^R_{\rho ij}, I^R_{\sigma ij}, I^R_{\tau ij} >\right\}$$

$Hence\ I^P_C \wedge_P I^Q_C \wedge_P I^R_C \in$ ICPFSM$_{(m \times n)}$

Property 5.2. Let $I^P_C, I^Q_C, I^R_C \in ICPFSM_{(m \times n)}, then$

(i) $I^P_C \vee_R I^Q_C \vee_R I^R_C \in$ ICPFSM$_{(m \times n)}$

(ii) $I^P_C \wedge_R I^Q_C \wedge_R I^R_C \in$ ICPFSM$_{(m \times n)}$

Proof:

$I_C^P = \{[<[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P], I_{\alpha ij}^P>, <[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P], I_{\beta ij}^P>, <[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P], I_{\gamma ij}^P>\}$

Here, $\underline{I}_{\alpha ij}^P \leq I_{\alpha ij}^P \leq \overline{I}_{\alpha ij}^P$, $\underline{I}_{\beta ij}^P \leq I_{\beta ij}^P \leq \overline{I}_{\beta ij}^P$, $\underline{I}_{\gamma ij}^P \leq I_{\gamma ij}^P \leq \overline{I}_{\gamma ij}^P$

$I_C^Q = \{[<[\underline{I}_{\mu ij}^Q, \overline{I}_{\mu ij}^Q], I_{\mu ij}^Q> <[\underline{I}_{\eta ij}^Q, \overline{I}_{\eta ij}^Q], I_{\eta ij}^Q> <[\underline{I}_{\xi ij}^Q, \overline{I}_{\xi ij}^Q], I_{\xi ij}^Q>\}$. Here, $\underline{I}_{\mu ij}^Q \leq I_{\mu ij}^Q \leq \overline{I}_{\mu ij}^Q$, $\underline{I}_{\eta ij}^Q \leq I_{\eta ij}^Q \leq \overline{I}_{\eta ij}^Q$, $\underline{I}_{\gamma ij}^Q \leq I_{\gamma ij}^Q \leq \overline{I}_{\gamma ij}^Q$

$I_C^R = \{[<[\underline{I}_{\rho ij}^R, \overline{I}_{\rho ij}^R], I_{\rho ij}^R> <[\underline{I}_{\sigma ij}^R, \overline{I}_{\sigma ij}^R], I_{\sigma ij}^R> <[\underline{I}_{\tau ij}^R, \overline{I}_{\tau ij}^R], I_{\tau ij}^R>\}$

$Here, \underline{I}_{\rho ij}^R \leq I_{\rho ij}^R \leq \overline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R \leq I_{\sigma ij}^R \leq \overline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R \leq I_{\tau ij}^R \leq \overline{I}_{\tau ij}^R\ all \in$ ICPFSM(m x n)

$(i) \Rightarrow max\{<\underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P>, <\underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q>, <\underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R>\}$

$\leq min\{<I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P>, <I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q>, <I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R>\}$

$\leq max\{<\overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P>, <\overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q>, <\overline{I}_{\rho ij}^R, \overline{I}_{\sigma ij}^R, \overline{I}_{\tau ij}^R>\}$

Now, we have, $I_C^P \vee_R I_C^Q \vee_R I_C^R = I_C^M \Rightarrow max\{<\tilde{I}_{\alpha ij}^P, \tilde{I}_{\beta ij}^P, \tilde{I}_{\gamma ij}^P>, <\tilde{I}_{\mu ij}^Q, \tilde{I}_{\eta ij}^Q, \tilde{I}_{\xi ij}^Q>, <\tilde{I}_{\rho ij}^R, \tilde{I}_{\sigma ij}^R, \tilde{I}_{\tau ij}^R>\}$

$$\leq min\{<I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P>, <I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q>, <I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R>\}$$

Hence $I_C^P \vee_R I_C^Q \vee_R I_C^R = I_C^M \in$ ICPFSM(m x n)

(ii) Let $I_C^P, I_C^Q and I_C^R \in$ ICPFSM(m x n)

$\Rightarrow min\{<\underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P>, <\underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q>, <\underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R>\}$

$$\leq max\{<I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P>, <I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q>, <I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R>\}$$

$$\leq max\{<\overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P>, <\overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q>, <\overline{I}_{\rho ij}^R, \overline{I}_{\sigma ij}^R, \overline{I}_{\tau ij}^R>\}$$

Now, we have, $I_C^P \wedge_R I_C^Q \wedge_R I_C^R = I_C^M$

$\Rightarrow min\{<\tilde{I}_{\alpha ij}^P, \tilde{I}_{\beta ij}^P, \tilde{I}_{\gamma ij}^P>, <\tilde{I}_{\mu ij}^Q, \tilde{I}_{\eta ij}^Q, \tilde{I}_{\xi ij}^Q>, <\tilde{I}_{\rho ij}^R, \tilde{I}_{\sigma ij}^R, \tilde{I}_{\tau ij}^R>\}$

$$\leq max\{<I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P>, <I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q>, <I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R>\}$$

Hence $I_C^P \wedge_R I_C^Q \wedge_R I_C^R \in$ ICPFSM(m x n)

Property 5.3

$Let\ I_C^P, I_C^Q, I_C^R \in ICPFSM_{(m\times n)}\ such\ that, max\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$
$\leq min\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\} for\ all\ i, j, then$
$R - Union\ of\ I_C^P, I_C^Q\ and\ I_C^R\ is\ also\ an\ ICPFSM_{(mxn)}$

Proof:

Let $I_C^P = \left\{< \left[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P\right], I_{\alpha ij}^P >, < \left[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P\right], I_{\beta ij}^P >, < \left[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P\right], I_{\gamma ij}^P >\right\}$,

Here, $\underline{I}_{\alpha ij}^P \leq I_{\alpha ij}^P \leq \overline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P \leq I_{\beta ij}^P \leq \overline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P \leq I_{\gamma ij}^P \leq \overline{I}_{\gamma ij}^P$

$I_C^Q = \left\{\left[< \left[\underline{I}_{\mu ij}^Q, \overline{I}_{\mu ij}^Q\right], I_{\mu ij}^Q >< \left[\underline{I}_{\eta ij}^Q, \overline{I}_{\eta ij}^Q\right], I_{\eta ij}^Q >< \left[\underline{I}_{\xi ij}^Q, \overline{I}_{\xi ij}^Q\right], I_{\xi ij}^Q >\right\}$

Here, $\underline{I}_{\mu ij}^Q \leq I_{\mu ij}^Q \leq \overline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q \leq I_{\eta ij}^Q \leq \overline{I}_{\eta ij}^Q, \underline{I}_{\tau ij}^Q \leq I_{\tau\gamma ij}^Q \leq \overline{I}_{\tau ij}^Q$

$I_C^R = \left\{\left[< \left[\underline{I}_{\rho ij}^R, \overline{I}_{\rho ij}^R\right], I_{\rho ij}^R >< \left[\underline{I}_{\sigma ij}^R, \overline{I}_{\sigma ij}^R\right], I_{\sigma ij}^R >< \left[\underline{I}_{\tau ij}^R, \overline{I}_{\tau ij}^R\right], I_{\tau ij}^R >\right\}$

Here, $\underline{I}_{\rho ij}^R \leq I_{\rho ij}^R \leq \overline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R \leq I_{\sigma ij}^R \leq \overline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R \leq I_{\tau ij}^R \leq \overline{I}_{\tau ij}^R all \in ICPFSM_{m\times n}$

$\Rightarrow max\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$
$\leq min\left\{< \tilde{I}_{\alpha ij}^P, \tilde{I}_{\beta ij}^P, \tilde{I}_{\gamma ij}^P >, \quad < \tilde{I}_{\mu ij}^Q, \tilde{I}_{\eta ij}^Q, \tilde{I}_{\xi ij}^Q >, < \tilde{I}_{\rho ij}^R, \tilde{I}_{\sigma ij}^R, \tilde{I}_{\tau ij}^R >\right\}$

From the hypothesis,

$\Rightarrow max\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$
$\leq min\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$

It follows that,

$\Rightarrow max\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$
$\leq min\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$
$\leq max\left\{< \overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P >, < \overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q >, < \overline{I}_{\rho ij}^R, \overline{I}_{\sigma ij}^R, \overline{I}_{\tau ij}^R >\right\}$

$Thus\ I_C^P \wedge_R I_C^Q \wedge_R I_C^R = I_C^M = is\ an\ ICPFSM_{(m\times n),}$

$If\ max\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$
$\leq min\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$

Property 5.4.

$Let\ I_C^P, I_C^Q, I_C^R \in ICPFSM_{(m\times n)}\ such\ that, min\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$
$\leq max\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$for all i, j then

R − Intersection of$I_C^P, I_C^Q\ and\ I_C^R\ is\ also\ an\ ICPFSM_{(mxn)}$

Proof:

Let $I_C^P = \left\{< \left[\underline{I}_{\alpha ij}^P, \overline{I}_{\alpha ij}^P\right], I_{\alpha ij}^P >, < \left[\underline{I}_{\beta ij}^P, \overline{I}_{\beta ij}^P\right], I_{\beta ij}^P >, < \left[\underline{I}_{\gamma ij}^P, \overline{I}_{\gamma ij}^P\right], I_{\gamma ij}^P >\right\}$,

Here, $\underline{I}_{\alpha ij}^P \leq I_{\alpha ij}^P \leq \overline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P \leq I_{\beta ij}^P \leq \overline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P \leq I_{\gamma ij}^P \leq \overline{I}_{\gamma ij}^P$

$I_C^Q = \left\{\left[< \left[\underline{I}_{\mu ij}^Q, \overline{I}_{\mu ij}^Q\right], I_{\mu ij}^Q >< \left[\underline{I}_{\eta ij}^Q, \overline{I}_{\eta ij}^Q\right], I_{\eta ij}^Q >< \left[\underline{I}_{\xi ij}^Q, \overline{I}_{\xi ij}^Q\right], I_{\xi ij}^Q >\right\}$

Here, $\underline{I}_{\mu ij}^Q \leq I_{\mu ij}^Q \leq \overline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q \leq I_{\eta ij}^Q \leq \overline{I}_{\eta ij}^Q, \underline{I}_{\tau ij}^Q \leq I_{\tau\gamma ij}^Q \leq \overline{I}_{\tau ij}^Q$

$I_C^R = \left\{\left[< \left[\underline{I}_{\rho ij}^R, \overline{I}_{\rho ij}^R\right], I_{\rho ij}^R >< \left[\underline{I}_{\sigma ij}^R, \overline{I}_{\sigma ij}^R\right], I_{\sigma ij}^R >< \left[\underline{I}_{\tau ij}^R, \overline{I}_{\tau ij}^R\right], I_{\tau ij}^R >\right\}$

Here, $\underline{I}_{\rho ij}^R \leq I_{\rho ij}^R \leq \overline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R \leq I_{\sigma ij}^R \leq \overline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R \leq I_{\tau ij}^R \leq \overline{I}_{\tau ij}^R all \in ICPFSM_{m\times n}$

$$\Rightarrow max\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$$
$$\leq min\left\{< \tilde{I}_{\alpha ij}^P, \tilde{I}_{\beta ij}^P, \tilde{I}_{\gamma ij}^P >, \quad < \tilde{I}_{\mu ij}^Q, \tilde{I}_{\eta ij}^Q, \tilde{I}_{\xi ij}^Q >, < \tilde{I}_{\rho ij}^R, \tilde{I}_{\sigma ij}^R, \tilde{I}_{\tau ij}^R >\right\}$$

From the hypothesis,

$$\Rightarrow min\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$$
$$\leq max\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$$

It follows that,

$$\Rightarrow min\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$$
$$\leq max\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$$
$$\leq min\left\{< \overline{I}_{\alpha ij}^P, \overline{I}_{\beta ij}^P, \overline{I}_{\gamma ij}^P >, < \overline{I}_{\mu ij}^Q, \overline{I}_{\eta ij}^Q, \overline{I}_{\xi ij}^Q >, < \overline{I}_{\rho ij}^R, \overline{I}_{\sigma ij}^R, \overline{I}_{\tau ij}^R >\right\}$$

$Thus\ I_C^P \wedge_R I_C^Q \wedge_R I_C^R = I_C^M\ is\ an\ ICPFSM_{(m\times n),}$

$$If\ min\left\{< \underline{I}_{\alpha ij}^P, \underline{I}_{\beta ij}^P, \underline{I}_{\gamma ij}^P >, < \underline{I}_{\mu ij}^Q, \underline{I}_{\eta ij}^Q, \underline{I}_{\xi ij}^Q >, < \underline{I}_{\rho ij}^R, \underline{I}_{\sigma ij}^R, \underline{I}_{\tau ij}^R >\right\}$$
$$\leq max\left\{< I_{\alpha ij}^P, I_{\beta ij}^P, I_{\gamma ij}^P >, < I_{\mu ij}^Q, I_{\eta ij}^Q, I_{\xi ij}^Q >, < I_{\rho ij}^R, I_{\sigma ij}^R, I_{\tau ij}^R >\right\}$$

V. CONCLUSION

In this paper, we construct the structure of internal cubic picture fuzzy soft matrices(ICPFSMs) with the synthesis of ICSM and PFM. Furtherly, we develop the union and intersectionof (P and R) order of ICPFSM and their related algebraic operations are discussed with suitableillustrations. From this, we can embark on the concept of anexternal cubic Picture fuzzy softmatrices in future.

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