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A General Solution for a Single Linear Equation by Theoretical Approach (Mathematical)

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ABSTRACT: This paper is fully about solving the linear equation in x and y by the method of sequence and using at least one solution to find the other required solutions. In this paper, I will be using a simple linear equation of form $ax + by + c = 0$ and will be proving the result.

THEOREM:

Let us consider a linear equation in the form $ax + by + c = 0$
where a, b, c are constants and are not simultaneously zero.
Here, let us assume that (x_1, y_1) be the one of the solution of the above linear equation, then the other solutions will be
$x = x_1 \pm b\,(n - 1)$ And $y = y_1 \pm a\,(n - 1)$ where $n \epsilon R$ and

$\{x_1 + b(n - 1), y_1 - a(n - 1)\}$, $\{x_1 - b(n - 1), y_1 + a(n - 1)\}$
are the two pairs of general solutions.

PROOF:

We consider a linear equation,

$ax + by + c = 0$ -------------- (1) Now,

Let us find any one solution to the given equation (1) and suppose we got (x_1, y_1) as a solution.

Similarly, we need to find at least two for our convenience and suppose we got (x_2, y_2) and (x_3, y_3) as two further solutions.

And if we continue the process of finding the values of x and y term, we get this type of sequence as mentioned. The sequence is in ascending order of solutions (i.e. $x_3' < x_2'$ and so on) and the sequence has to be in either all Integers, fractions or Irrationals.

For X;

$S_0 = \ldots \ldots \ldots \ldots \ldots \ldots , x_3', x_2', x_1', x_1, x_2, x_3, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$

Here, x_1', x_2', x_3' are the respective solutions of linear equations for the variable x.

The above sequence will be a diverging sequence from x_1 on both directions.

So, let's contract the sequence to the right of x_1;

$S_{x1} = x_1, x_2, x_3, \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$

Here, the above sequence will be in an A.P.

Hence, the common difference between the terms is given by

$d_1 = x_2 - x_1 = b$----------- (2)

If we include the sequence to the left of x_1 i.e. $x_1', x_2', x_3', \ldots$..

Then,

Common difference will be,

$d_2 = x_2' - x_1' = -b$--------- (3)

As a whole we get,

Common difference $(d) = \pm b$

So using the laws of A.P. we get,

$t_n = a_1 \pm b\,(n-1)$---------- (4)

Taking a_1 (first term) as x_1 we get to the following formula for x;

x = $x_1 \pm b\,(n-1)$------------(5) [Where, d = $\pm b$]

For Y;

Quite similarly, we can find the result similar to as x;

$S_0' = \ldots \ldots \ldots . \ldots \ldots . . \ldots, y_3', y_2', y_1', y_1, y_2, y_3, \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$

Here, y_1', y_2', y_3' are the respective solutions of linear equations for the variable y.

Using similar process as for x, we get to the following conclusions;

Common difference$(d) = \pm a$

First term (a_1) = y_1

Using the laws of A.P. we get,

$t_n = a_1 \pm a\,(n-1)$

$y = y_1 \pm a\,(n-1)$ ------------ (6)

Hence, we get to the general solution of the given linear equation which is as mentioned in the following ordered pairs.

$\{x_1 + b(n-1),\ y_1 - a(n-1)\}$, $\{x_1 - b(n-1),\ y_1 + a(n-1)\}$

Note:

1) The value of x which we get by taking the positive sign in t_x i.e. $x_1 + b(n-1)$, this will be in the pair with the value of y which we get from taking negative sign in t_y i.e. $y_1 - a(n-1)$

2) *Why does $x_2 - x_1 = \pm b$ and $y_2 - y_1 = \pm a$?*

For this we consider,

(x_1, y_1) and (x_2, y_2) be any two solutions of the linear equation $ax + by + c = 0$.

Then,

$ax_1 + by_1 + c = 0$ -------------(7)

$ax_2 + by_2 + c = 0$ -------------(8)

Changing both equations in terms of x_1 and x_2 respectively we get,

$$x_1 = {-(by_1 + c)}/{a}$$

$$x_2 = {-(by_2 + c)}/{a}$$

Subtracting x_1 from x_2 ,we get

$$x_2 - x_1 = \frac{-(by_2+c)}{a} - \frac{-(by_1+c)}{a}$$

$$= \frac{-by_2-c+by_1+c}{a}$$

$$= \frac{b(y_1-y_2)}{a}$$

$$= \frac{b(\pm a)}{a} [(y_2 - y_1)=\pm a]$$

$$= \pm b$$

And by similar method we can prove that $y_2 - y_1 = \pm a$ taking into consideration $x_2 - x_1 = \pm b$.

II. CONCLUSION:

Hence, the above mentioned theorem can be used to find out solutions to the single linear equation in x and y quickly. The aforementioned proof will let us ponder into the topics related to linear systems and through this we can also find the complex roots of the linear equation.