

How to make mathematics love learners?

¹ARMAND, ²Ravelokoto Hirijha-Velo, ³André TOTOHASINA

¹High School Antsiranana B.P. 234 Antsiranana Madagascar.

²Higher Polytechnic School Antsiranana University of Antsiranana

³Department of Mathematics and Computer Science

Superior Normal School for Technical Education (S.N.S.T.E.)

University of Antsiranana B.P.0 - Antsiranana Madagascar

ABSTRACT: The fact that most of the learners mostly disdain scientific disciplines, especially the mathematical discipline, UNESCO has already launched a challenge on the teaching of mathematics at the international level. The contribution to this challenge is highly desired given its size. Indeed, this teaching sector suffers from numerous problems for all disciplines. We present here some contribution about this "challenge". The next step is to explore the basis of these disdain in order to recommend effective solutions to UNESCO's recommendations. In this respect, writing skills, communication and mathematical modeling hold a privileged and important place in mathematics education. Thus, this paper is intended to give some recommendations about writing, communication and mathematical modeling.

KEY WORDS: Mathematical writing, teachers, learning, mathematical demonstration

I- INTRODUCTION

Madagascar's education system is in the process of being reformed with a view to improving efficiency in all subjects (UNESCO, 2011). Thus, the school program book gives a good instruction or directive to teachers "the teaching of mathematics aims to develop in students a spirit of rigor and objectivity so as to make it able to open and act on the concrete, complex and diversified world ". Mathematics is a compulsory subject to be taught from the primary to the final year, regardless of the series or technical specialty. It is an essential basic subject in the formation of a future citizen and in education (UNESCO, 2011), (M. Wambst& Y. Genzmer, 2008), (André Totohasina, 2011). More specifically, we are interested in seeking out and recommending solutions that can remove this widespread misunderstanding of the discipline. Moreover, it is not a mistake to say that the study of mathematics has also become a fundamental tool in the development of human intelligence. Over the years of our work on the teaching of mathematics, we have observed that learning the disciplines applied to solve mathematical problems still raises difficulties of misunderstanding. Indeed, the writing used by the students during the resolution of their homework, either on the board or copy sheet unfortunately do not appear illegible and incomprehensible. Each school year, we provide tutoring to trainees who are fifth-year students of the Superior Normal School for Technical Education (S.N.S.T.E.).

This allowed us to collaborate with expert teams of Research Teachers responsible for monitoring and supervising student trainee teachers: at the end of the trainee's observation session, the last 45 minutes, each of three or four Teachers researchers in terms of constructive criticism or approval of observed behavior sometimes. This fact prompts us to analyze in greater depth methods of improving mathematics learning. We will therefore propose solutions to illustrate or concretize.

II- OBSERVATIONS AND PROBLEMS

Currently, no one is unaware of the growing disaffection of the mathematical discipline by learners. The reasons are diverse. Underdeveloped countries like Madagascar have, apart from common reasons, specific and specific reasons. The obvious common reason is that this discipline is often thought to be too abstract and, therefore, easily deviates from direct practical application, hence the tendency towards the idea of automatic rejection, i.e. to say negligence.

To this end, the PISA report of recent years informs us that students in Singapore's education system are the best in the world in mathematics at the primary and Middle School levels. The Singapore's method (SM) on the teaching of mathematics proves thus effective, if not the most effective among the pedagogical methods existing in the literature such as the Pedagogies by Objective (PPO), the Explicit pedagogy (ExP), the APC approach by Competence, etc.. Finally, this SM rests on a pedagogical approach qualified as Concrete-Imaged-Abstract Approach (CIAA) (Jean-Michel Jamet, 2011) evolving in the three following stages:

- i. 1st step: concrete step Students are first confronted with mathematical notions through the manipulation of objects with initiative: it involves involving learners in their learning.
- ii. 2nd step: pictorial step this time, the manipulated objects are replaced by images that represent them. Now, students must verbalize to identify and interpret these appropriate images as well as invariants; it is a semi-concrete and semi-abstract stage.
- iii. Finally, 3rd step: step of abstraction.

Symbols and explicit mathematical formalism are introduced. Then, reinforcement exercises will have to contain problems of skins diversified in contexts. From our point of view, the SM thus proves to be a composite pedagogical approach, that is to say a combination of APC and ExP, that is to say APC followed by ExP and not the opposite. Like the formula $APC = PPO + \text{socioconstructivism}$, we symbolize this by the methodological equation: $SM = APC + ExP$, the addition not being commutative here. Indeed, the learner's call to action optimizes the desire to learn and thus produces true learning. A teaching based on the resolution of the problem situations and the involvement of the learner has a great chance to lead the student to understand the usefulness of mathematics and to love this subject. The teacher of mathematics should then wear his toga of pedagogical leader, given his class-group: he must deploy his charismatic leadership and transformational leadership. This is our firm conviction on this noble but difficult task of teaching mathematics. As a result, in our personal opinion, presumably, one of the significant reasons for rejection of the discipline is the following: often even mathematics teachers (not to mention all science teachers) and/or society forget to do to learners Praise or publicity of the discipline they teach, through the story of the exploits that are crossed by the discipline for example (i.e. it is a question of knowing how to make one's subject love). A third reason, and this one is very specific for the poor countries (with mainly low-income population like Madagascar), is the lack of motivation or stimulus emanating from the parents of pupils themselves. Indeed, in meetings with the parents of students, one hears almost continuously this chorus: "it is better to reinforce the literary disciplines, because the latter easily lead to functions or posts with comparatively heavy monthly income, to a President of court by example. In a word, scientific disciplines lose their values (Keith Devlin, 2012), (Kevin Houston, 2009). The following question arises: "If everybody is in a hurry to become president of the court, what is the outcome? The question is there, and all of us must answer it. The idea comes from us, mathematics teachers from Middle School and high schools and always remember that since then our educational activities, and this, in classes of different levels, the mistakes made by students when solving mathematical problems instead of disappearing as they go, they only evolve in the negative direction. This finding is frequently raised in discussions with colleagues during surveys of learners and parents about their relationship to the mathematical discipline. According to statistics from all educational institutions and institutions in Madagascar, the numbers of graduates and learners of literary series far exceed those of the scientific series. In this regard, specialists are particularly worried about the lack of interest of learners in scientific disciplines, especially mathematics. In Madagascar, for example, the statistics on baccalaureate candidates on the website of the Ministry of Higher Education have stated that the percentage of candidates enrolled in the scientific series is not even exceeded by 6%. The last nine years and it has dropped to 4% for the 2016 and 2017 sessions. Presumably, society or even parents and teachers encourage learning to quit such demotivation. If we ask to summarize, we will simply say: "mathematics have lost their value: the malicious have pushed back. Mathematics being at the base of all sciences, the affirmation will be equivalent to: the whole scientific disciplines are collapsed! ". Fortunately, we have already asked the question above, and everyone has to find a reasonable answer! However, the commonly recognized, reliable and well-founded solution for this topic remains to be found. We already think that it is likely to be positively correlated with minimal mastery of mathematical reasoning; however this remains to be confirmed and elucidated. This is our problematic in didactics of mathematics. We believe that the didactical advantages resulting from the present research are, among others, to be able to identify and remedy the errors of mathematical writing made by middle and High School students in Madagascar or elsewhere, compared to the challenges of Mathematics teaching currently launched by UNESCO (UNESCO, 2004), (UNESCO, 2011).

III- MATHEMATICAL SIGNIFICANCE OF THE DIFFERENT WORDS FREQUENTLY ENCOUNTERED IN MATHEMATICAL QUESTIONS AT MIDDLE SCHOOL AND HIGH SCHOOL

There are many ways to write. You must take note for your wish and future employment to communicate or expose an idea to another. Whatever the reason, writing mathematics is a difficult art and requires the practice of producing clear and effective work (Kevin Houston, 2009), (EduSCOL, 2009), (Kolette E. & Albert, I. Calin, 1993), (D. Delaruelle & L. Misset, 1994), (Jules Payot, 1913), (G. Polya, 1965), (Cabassut Richard, 2005), (Gerard Dumont, 2004), (Stéphane Enrich, 2209). It should be noted that all questions used in all disciplines are also found in Mathematics. However, it should be noted that some groups of words are frequently repeated when mathematical questions are set up in high schools and Middle School and many others. Their meanings are

certainly found in dictionaries such as Larousse, Robert, etc., but it turns out to indicate here their particular meanings in the field of mathematics. The question therefore comes down to: When do we use the words: "interpret, explain, prove, demonstrate, show, discuss, verify, justify, establish, calculate, determine, form, formulate, deduce, find, factor, simplify, solve, specify, define, express, train "in mathematics? This paper will attempt to answer it.

3.1. To interpret

The question containing the word "to interpret" in mathematics is not practicable at the Middle School level. Indeed, Middle School students do not yet practice highlighting all the theorems or properties that were assigned to them when solving mathematical problems. In mathematics, however, the answer to questions requires above all the application of personal knowledge (highlighting, ownership, etc.). Therefore, this question needs reasoning based on the personal analysis of the character of the elements or events to be studied. In high school, for example, this word is often used in the graphical and statistical interpretation of the variance, expectation and linear correlation coefficient, etc.

Example 1. Study on consumer goods in France, from 1984 to 1992 (D. Delaruelle, 1994). The two characters studied are imports and exports expressed in billions of francs.

Year 19...	84	85	86	87	88	89	90	91	92
Import x_i	117	128	139	153	170	194	208	215	215
Export y_i	114	125	123	128	143	166	176	181	188

1. Using this table, "show that the covariance of this statistical series is 961.3".
2. Using this same table, "show that the linear correlation coefficient r is 0.9832", "Interpret" the result.

3.2. To explain

The word is used when the candidate is asked to make the audience understand an "object" that is still considered obscure, this object being a notion, a (physical) phenomenon, a new idea, etc. The *raison d'être* of the "thing" and the reason for its existence would be clear after the completion of the operation. Subsequently, it is clear that all the tool-words used as basis for this operation (theorem, definition, axiom, proverb, corollary, lemma, etc.) are included in the solution of this problem itself. In mathematics, the word often refers to the term "specify" from the point of view meaning. In Middle School and high schools, the word is often followed by the other two following terms:

- Or explain "why»...?"
- Or explain "how»...?"

Example 2. 1. Explain why the number 2 is prime?

2. Explain how to get a factor decomposition?

3.3. To prove

The proof in mathematics, classically receives the name of "demonstration" and meets specific writing standards that make it a very singular kind of speech. This characteristic largely contributes to the idealization of the final rigor that some envy. Yet within the math community, things are a bit different. That is to say, in mathematics, the word "prove" is used when the way to be used to solve the problem passes through several stages of demonstrations or properties before reaching the desired result.

Example 3. In an equilateral triangle of side a , "prove that the height is equal to $\frac{a\sqrt{3}}{2}$ ".

Indeed, to answer the question posed, the property of Pythagoras on a right triangle at a point, the theory of height, the idea of axis of symmetry and the notion of the equation $x^2 = k$, where k - positive realities are necessarily among the tools to use.

3.4. To demonstrate

In mathematics, the use of the word "demonstrate" refers to the use of at least one theorem, a property, etc. seen in lesson, and that one wishes used by the candidate during the resolution of the problem.

Example 4. If there is a right triangle ABC in A, and we want to make sure that this triangle is actually a rectangle in A without the way to follow is indicated, while we want that the learner passes precisely by this

way, we put the word "demonstrate". Recall that, in the field of mathematics, there are several modes of demonstration or reasoning namely for example the reasoning by: "Direct application of the theorem" (it is a mode of reasoning most used especially at Middle School level), Contraposed, The absurd, Analysis-synthesis, Counter-example, Case disjunction, Truth table, Recurrences etc. We see this the links of the four words defined above according to their dependencies to the world of mathematics.

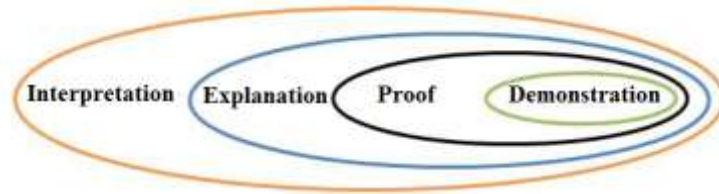


Figure 1 – Classification of four words according to the degree of dependence

3.5. To show

Here, the goal is approximately the same as in the previous case, but only the question asked is less demanding, and therefore more flexible, that is to say, the resolution tools are previously explained.

Example 5. The question on the right triangle above will be as follows: "using the Pythagorean Theorem, show that ABC is a rectangle in A". Take a look at the two questions on the study of current consumption in France, from 1984 to 1992. If we want to answer this question, nothing to prove and nothing to prove but just use the table and apply the expression of covariance and we perform the calculation.

3.6. To discuss

In general, the question that contains it is not practiced in Middle School except at the 4th and 3rd level to the construction of the equation line $ay + bx + c = 0$ because, if $a = 0$ and $b \neq 0$ then, this right is horizontal; if $b = 0$ and $a \neq 0$ then this line is vertical. Subsequently, this word is used when the answer of the question which contains it requires the use of a reasoning based on the disjunctions of the cases. In mathematics, the question that contains it, in general, is followed by the word "following".

Example 6. Discuss according to the value of the real m the solution of the equation:

$$mx^2 + 2(m + 3)x - 4 = 0 \text{ in } \mathbb{R}.$$

Discuss according to the value of the real λ the limit of the function f , in $\pm\infty$ defined by: $f(x) = \frac{\lambda x + 4}{x + 3}$.

3.7. To check

The word check is used to say that it will be necessary to establish a relation existing between two different expressions of forms but equal quantitatively. The question asked is often followed by the relative pronoun "that"

Example 7. Check that for every a, b of $\mathbb{R}$ we have: $(a + b)^2 = a^2 + 2ab + b^2$.

3.8. To justify

We use the word when we are in the presence of an affirmation whose root or purpose is sought. The question asked is often followed by the relative pronoun "que".

Example 8. Justify that in $\mathbb{R}$ every equation of the form $x^2 + a = 0$ ($a > 0$) does not admit any root.

3.9. To establish

The word requires a well-defined expression from the initial data.

Example 9. Given two lines (D1) and (D2) intersecting at a point A. The points B and C are two points respectively belonging to (D1) and (D2). The parallel to (BC) is (MN), with M and N respectively belonging to (D1) and (D2) and such that $M \neq B$ and $C \neq N$. The appropriate word for the question on the relationship between the areas of ABC and AMN triangles are here: "Establish" the relation between the areas of triangle ABC and AMN.

3.10. To calculate

Is to use when the answer requires operations done manually or mentally.

Example 10. Calculate the number $987 - 45689$

3.11. To determine

The word requires a literal expression or a numeric value as a result of the operations performed by a few steps.

Example 11. Let f be the function defined by: $f(x) = \frac{x+1}{2x+3}$.

At High School level "Determine" the limit of f to infinity;

At the Middle School level "determine" the definition set of f .

3.12. To form

At the level of Middle School and High Schools, the word has almost the same value as "establish".

Example 12. See the example on "establish".

3.13. To formulate

The word is used to improve guidelines for teaching. Often, his job matches that of the word "express", in the final hope of obtaining a formula or an expression.

Example 13. Formulate your preparation well so that you do not cheat during your course.

3.14. To deduce

The question using "Deduce" is placed after the statement or the obtaining of a concrete result which exposes in another implicit and apparently obvious way another result. The term connects two objects: an antecedent or cause, and an image or consequence.

Example 14. Let x be a real number such that $3x = 90$ deduce the value of x .

It should be noted that, for any field, all students must also have the deductive spirit in order to quickly develop their knowledge or knowledge. Indeed, it constitutes as base and / or founder to face the resolutions of the new problems among said pupils.

3.15. To find

Its use is very common at the Middle School level, given its obvious clarity. However, the answer that follows requires a numerical value. The question corresponding to this word is often independent.

Example 15. Can we "find" a real number x such that $x^2 = -5$.

3.16. To factories

It is asked to write the given expression as a product of the factors.

Example 16. Factorize the number 10.

3.17. To simplify

We ask for a simpler expression than the initial expression. At High School and Middle School, we can meet three different places of frequent use of the word: Fraction, Algebraic Sum, and Factor Product.

Example 17. Simplify writing 10^{-6} , 130000000 etc.

3.18. To resolve

The word "solve" and the question that contains it are dear to mathematicians because they represent indeed "objects" specific to mathematics. Recall here that a root of an equation is any value that satisfies the given equation. But a solution of an equation is the root that verifies the equation only in a well-defined set. It should be noted that in middle and High School mathematics, we encounter several particular sets of numbers, namely: $\mathbb{N}, \mathbb{Z}, \mathbb{D}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$ and all their subsets. And the objective of the question is precisely the verification of the distinctive knowledge by the learners of the aforementioned sets.

Example 18. The question " Solving the equation $x + 3 = 0$ " is clumsy because no work set is specified here. It will be necessary to write: " Solve in $\mathbb{N}$, or in $\mathbb{Z}$ the equation $x + 3 = 0$ ", for example.

3.19. To build

We ask here the graphical representation of a figure. The term has the same value as tracing.

Example 19. Construct on a graduated line of the set $\mathbb{N}$.

3.20. To give

We use the corresponding question when we do not require an analysis followed by operations, but just an expression or a definition given in class.

Example 20. Give the diagonal expression of a side square a according to its side.

3.21. To specify

The word implies the prior existence of a consequent affirmation and / or result, but the explanations that led to these affirmations and / or results are still unsatisfactory and unconvincing to the subject for which they are intended. The latter may then ask for additional details.

Example 21. On the equation $2x + 3 = 0$ a student claims that a solution is $-\frac{3}{2}$. Here the answer does not give any indication about the whole work, it is indeed unacceptable. Then it will be necessary to ask this student the question: To specify in which sets the solution is valid.

3.22. To define

We use it when we ask for a definition. In general it is a question of court.

Example 22. Define a distribution function of a random variable X .

3.23. To study

It's a very standard question. Indeed, the word requires activities of very different natures and can be of various domains. We use this word when we ask to examine thoroughly and attentively, to dissect a problem.

Example 23. Study the variation of the function f .

3.24. To express

This is a question that requires an expression.

Example 24. In a square, express the diagonal according to its side.

3.25. To train

It is a vocabulary used especially if one wants to obtain the result in table form.

3.26. Comments

We have explained these different words because, the knowledge and in-depth practice of these words through mathematical activities brings learners to a feeling of affection and deep attachment to the understanding of mathematics. For a good reason, even Singapore's education system is the world's best in elementary and Middle School mathematics involving the application of the Concrete-Imaged-Abstract Approach (CIAA), yet at the level of these institutions especially in mathematics education it There are always some things that are uncreatable. Indeed, on the one hand, if one makes for example the subtraction of the five sticks including three oranges and two black ones and by removing the three oranges figure 2.

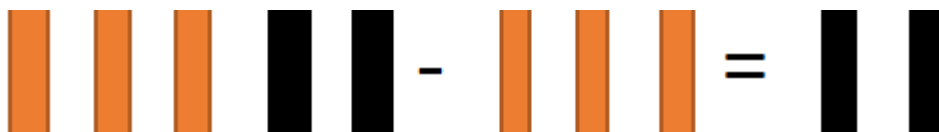


Figure 2 – Fivesticks

Indeed, all learners have no problem to have the two remaining black sticks. Thus, the subtraction of $5 - 3 = 2$ does not pose problems of concretization. On the other hand, at the Middle School level, "how to concretize the existence of negative numbers $3 - 5 = -2$ for example? ". Myself, I have no idea as a teacher of mathematics at

the level of general education High School except that it brings back to the mind of the learners the fact that they already know "abstractly" the rules of calculations algebraic signs months and more and we take the calculation on the subtraction above but this time by removing the negative sign and we have $3-5 = -(-3+5) = -(5-3) = -2$. In a real way, if we reflect on the epistemological reflection of the negative numbers that attached a lot to debt schemes through the loan-bank, these courses are not yet at their levels. In any case, probably, these courses can't yet reveal in a concrete way the existence of negative numbers. We believe that in such a case "the Concrete-Imaged-Abstract Approach (CIAA) becomes the Quasi-Concrete-Imaged-Abstract Approach (QCIAA).

To overcome this problem, the teacher must be a man imbued with a minimum of daring. This is why UNESCO has launched a qualitative challenge as a trainer, educator, teacher, etc.

IV- ANSWER TO MATHEMATICAL QUESTIONS

Recall that in general, there are four types of sentences including:

- Interrogative sentence: it is used to ask a question. It ends with a question mark (?).
Construction: Verb + Subject + Complement?
Question word + Verb + Subject + Complement?
- Exclamatory sentence: It is used to express emotions such as joy, pain, admiration, anger ...The exclamation sentence ends with an exclamation point (!).
Construction: Subject + Verb + Complement!
- Imperative or injunctive sentence: it serves to give an order, a council or to express a prohibition.
- The imperative sentence ends with an exclamation point (!) Or a period (.).
Construction: Verb + Subject + Complement (!) Or period (.).
- Declarative sentence: it serves to give information on a situation. We can describe an action, a landscape ...The declarative sentence ends with a period (.). Construction: Subject + Verb + Complement.

There are various ways of answering questions in the literature, depending on the form of the question asked and the nature of the environment in which it is to be answered.

It would also be better to indicate that there are two different basic ways of asking a question:

- The question constituted by "interrogative sentence"
- The question constituted by "imperative sentence"

If the answer to the question is in public, the imperative sentence gives preference. In fact, intercommunity respect is required here in the common search for the resolution of the problem.

Similarly, if the answer to the question is in the presence of the author, the imperative sentence still finds its proper place. On the other hand, if the problem is drawn by a stranger, and is to be corrected by a stranger, the declarative sentence is the best when it is written, the imperative sentence being recommended to exclude because a deep respect is no longer here. Required at the level of the corrector; the answer to the question is open to the general public, national or even international. Any student, or candidate, must dominate this process, even if he does not find the right answer, which is not mandatory in the question area. Indeed, everyone must know the way "how to respond", and no relationship is here to establish the nature of the answer (just or not).

Example25

Either answer the following questions

1. Solve in N the equation $2x + 6 = 0$.
2. Establish the relationship between the number 6 and 3.
3. Study the variation of the function f
4. For what value of x for x^2 to be equal to 4?
5. What are the solutions of the equation $x^2 - 9 = 0$.

All these questions, except those in interrogative form, accept modes of response not very far apart. Take, for example, questions 1, 2, 3. In general, we have three ways to write the questions based on the nature of the audience.

- In a classroom, that is, in the presence of the teacher asking the question: The imperative sentence is highly recommended here. Indeed, mutual respect must reign in a classroom, especially respect for the teacher who is sought together to show knowledge what he has transmitted to learners. Then the drafting will be to present as follows:

1. Solve in $\mathbb{N}$ the equation $2x + 6 = 0$.
2. Let's establish the relationship between the number 6 and 3.
3. Let's study the variation of the function f .

- In the competition or special session room, that is, we do not know who asked the question and who will correct the copies.

Here, the honor of the corrector is not of major importance, it is rather the respect that one attributes to the subject and its value that one seeks to make appear, while following a marked rigor during the writing. The imperative sentence is bad here. It will rather answer this way:

1. Resolutions in $\mathbb{N}$ of the equation $2x + 6 = 0$.
2. Establishment of the relationship between the number 6 and 3.
3. Study of the variation of the function f .

Or

1. We will solve in $\mathbb{N}$ the equation $2x + 6 = 0$.
2. We will establish the relation between the number 6 and 3.
3. We will study the variation of the function f .

The last two questions have the same mode of response for the "classroom" and "competition hall" environments, only the answer must conform to the need of the question

Example 26

1. For what value of x for x^2 to be equal to 4?
2. What are the solutions of the equation $x^2 - 9 = 0$?
 - ✚ It is awkward to write: "for $x^2 = 4$, $x = 2$ ".
 - Write instead "For $x^2 = 4$ ", it suffices that $x = 2$. Because it is a question that requires a sufficient condition.
 - The solutions of the equation $x^2 - 9 = 0$ are: -3 and 3.

V- HOW TO ADDRESS THE MATHEMATICAL WRITING

We have already explained in paragraph 3 the meaning of the various words frequently encountered when establishing the mathematical question. Indeed, the mastery of the mathematical meaning of these words makes it easier to follow the logic of mathematical writing when solving mathematical problems. Recall that, in general, we have four objectives to achieve to write a duty, namely: "master the architecture of reasoning, master the logical links articulating the reasoning, advance the good arguments and justify the delicate passages", which are valid whatever the discipline. In addition, a mathematics assignment and many other disciplines are well written when all the reasoning is complete (unambiguous or flawless and the text and / or the speech are very convincing for the partners) and can be understood by anyone. Which person knows the math program required effortlessly from them? Good writing is not synonymous with long writing! A good copy should contain only the necessary arguments, without redundancy or useless sentences. Subsequently, the mathematical writing is like sport, music etc., it needs daily practical training and at any time, because we have several morphemes to structure the mathematical writing (see conclusion). We will now give some guidelines for mathematical writing. In the following, we always consider ourselves in the classroom with the teacher or others. And then we will answer in paragraph 6 the parts of the questions in paragraph 3.

❖ How to write the reasoning?

First, for each question, the writing must include at least three parts including: introduction, reasoning, conclusion. Everyone is able to respect the introduction and conclusion points, just think about it. Writing the introduction allows you to appropriate the subject and its context. Writing the conclusion makes it possible to memorize more the result. In addition, a well written introduction allows the editor to read the copy and understand what it is without having to refer to the statement (which is all the more enjoyable!). And then you have to follow the following directions.

- Specify the number of the question treated, respecting the numbering of the statement.
- Introduce all the variables used, even if they are defined in the statement.

Example 27. Here are several ways to introduce a non-zero real:

Let $x \in \mathbb{R}^*$ or for all $x \in \mathbb{R}^*$ or else $\forall x \in \mathbb{R}^*$ or ...

Indicate the assumptions clearly and succinctly in order to define the basis of the reasoning.

Warning! It is not a question of copying the statement!

- Announce the steps of reasoning (for better readability).

Example 28.

- For a simple reasoning: Let's show ...
- For a complex reasoning: Let's show first ... It remains only to show ...
- Specify (if applicable) the reasoning method used.

Example 29

- Let us reason by recurrence, Reason by the absurd ...
- Highlight the logical articulations of reasoning and use wisely:
- ... so ..., if ... then ..., if and only if ..., from ... (Almost used to the last statement) or as a mathematical symbol)...

Note 1.

- An equation or inequation resolution requires equivalence reasoning, but otherwise, in general, the implications are sufficient. So, in this case, avoid marking equivalences that you do not justify and that are often false!
- Justify all statements by referring to the course (theorem, definition ...) or the result of an earlier question.

This is the most important point of writing. The reference to the course or the result of a question must be made with absolute precision, by checking and gathering all the necessary hypotheses before concluding.

Do not forget to quote the name of the theorem or definition used; the number of the question used...

Note 2.

- You can use a previous question even if you have not demonstrated it!

VI- REPLY TO SOME EXAMPLES OF PARAGRAPH 3

1. Explain why the number 2 is prime?

Let's explain why the number 2 is prime. All students must know how to construct this sentence. It is not a question of making a recitation, but only they must know its logical continuation.

We have many ways to approach our explanation work.

Either we use the definition of a prime number or in the end we draw the conclusion.

Either we study the characteristic of number 2 through a prime number and draw after the conclusion. We recommend two correct answers.

1. By definition, a positive integer n is said first if and only if its only divisors are 1 and n . If, for example, we denote by P the set of positive prime numbers, then according to the previous definition 2 is indeed element of P . Hence 2 is prime.

2. We can invert, as a continuation, this sentence: We know that the natural integer 2 has no divisor than 2 and 1 that is to say, it is exactly the sufficient and necessary condition for a number is prime. Therefore we can deduce that the number 2 is prime. Of course, the two answers above are not the only ones possible.

Subsequently, by grouping the aforementioned procedures, we have: explain "why the number 2 is prime" (introductory sentence) + by definition, a positive integer n is said first if and only if its only divisors are 1 and n . If for example we denote by P the set of positive prime numbers, then according to the previous definition 2 is indeed element of P (sentence which carries the reasoning to convince the repositories of the subject) + From where 2 is prime (the conclusion always affirms the general objective of the subject, which comes out in well-reasoned reasoning). That is, "Let's explain why the number 2 is first. By definition, a positive integer n is said first if and only if its only divisors are 1 and n . If, for example, we denote by P the set of positive prime numbers, then according to the previous definition 2 is indeed element of P . Hence 2 is prime".

Now we will answer the example 2. In an equilateral triangle of side a , "prove" that the height is equal to $\frac{a\sqrt{3}}{2}$.

Here, our goal is to have the height of a triangle equal to $(\frac{a\sqrt{3}}{2})$ or to know where the origin of the quantity $(\frac{a\sqrt{3}}{2})$ comes from. Let us first note the quantity to prove by "h", for example. Since we still have no direct relation or theorem to establish equality between h and $(\frac{a\sqrt{3}}{2})$, it would be better to introduce here some intermediate ways.

As we have seen above, the question "prove" always requires several intermediate demonstrations before reaching the desired result. Let ABC be an equilateral triangle of side a (see figure below)

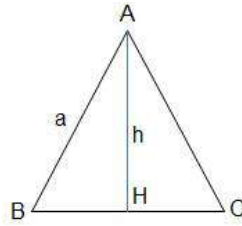


Figure 3 – Equilateral triangle ABC

We have:

$$AB = BC = CA = a.$$

Let H be the orthogonal projection of point A by the mediator of segment $[BC]$.

So, it is clear that the triangle ABH is a right triangle in H .

So, using the property of Pythagoras on a right triangle at a point (H for example), we have: $[AB] \ AB^2 = BH^2 + HA^2$, i.e. $AB^2 = BH^2 + h^2$ because $HA = h$.

$$\text{So,} \quad h^2 = AB^2 - BH^2 \\ = a^2 - BH^2.$$

$$\text{But, } BH = HC = \frac{a}{2} \text{ so, } BH^2 = \left(\frac{a}{2}\right)^2.$$

So,

$$\begin{aligned} h^2 &= a^2 - \left(\frac{a}{2}\right)^2 \Leftrightarrow h^2 = a^2 - \frac{a^2}{4} \\ \Leftrightarrow h^2 &= \frac{4a^2}{4} - \frac{a^2}{4} \\ \Leftrightarrow h^2 &= \frac{3a^2}{4} \end{aligned}$$

$$\Leftrightarrow h^2 = \pm \sqrt{\frac{3a^2}{4}}.$$

Since the distance is always positive; therefore,

$$h^2 = \pm \sqrt{\frac{3a^2}{4}} \Leftrightarrow h = \frac{a\sqrt{3}}{2}.$$

Hence the result

6.1. Comments

Usually, the mathematician always strives to look for simple mathematical relational objects in so-called complicated "objects" and everyday facts of human life. As a result, it has provided many advantages over mathematical notations represented by mathematical symbols, namely: equality ($=$), addition ($+$), for reals; ($\cup$) for sets; AB in geometry for the distance between point A and point B etc. However, the learner, the author and the user of the symbol and / or the notation used in mathematics must know how to write letter after letter said symbol and / or notation. We underline this so as not to distort the rigor and / or the mathematical connection then of the writing. And the teacher and the learner who solve this problem, that is, who write on the chalkboard or on the notebook the symbols that are evoked, must utter them point by point from their mouths.

Example 30. Taking the modelization above concerning the training of the demonstration so that a triangle is equilateral we have: either ABC an equilateral triangle of side a , that is to say that the distance between the point A and the point B is equal to the distance between point B and bridge C , is equal to the distance between point C and point A and is equal to a ; let H be the orthogonal projection of point A by the mediator of segment $[BC]$.

It is therefore clear that the distance between the point B and the point H is equal to the distance between the point H and the point C , is equal to a divided by two; in addition, the triangle ABH is rectangle at the point H . Then, using the property of Pythagoras at the right triangle at a point (H for example), we obtain: the distance

between the point A and the point B with the square is equal to the distance between point B and point H squared plus the distance between point H and point A squared, ie the distance between point A and point B squared is equal to the distance between point B and point H squared plus height h squared. So, having this relation, we can directly write: the height h to the square is equal to the distance between the point A and the point B squared minus the distance between the point B and the point H squared, is still equal to the distance a squared minus the distance between point B and point H squared. Now, we have at the beginning: the distance between the point B and the point H is equal to the distance between the point H and the point C, is still equal to the distance between the point B and the point C divided by two, and still is equal to the distance divided by two. Therefore, nothing prevents us from having: "the height h squared is equal to the distance a squared minus the distance divided by two the squared", which is equivalent to: "the height h squared is equal to the distance a squared minus the distance a squared divided by four ". Returning to the same denominator, we obtain: "the height h squared equals four times the distance a squared divided by four minus the distance a squared divided by four", which is equivalent to: "the distance h to square equals four times the distance a squared minus the distance a squared the whole divided by four ", which is still equivalent to: "the height h squared equals three times the distance a squared divided by four Because four minus one is equal to three. Starting from this relation we can deduce: the height h squared is equal to plus or minus square root of three times the distance to the whole divided by four. *This height h being indeed a distance, it can in no case be negative. The negative part does not represent the height, that is to say, the height h is equal to the square root of three times the distance a squared divided by four. Since the square root of quotient is equal to the quotient of the square root, so the height is equal to the square root of three times the distance a squared divided by the square root of four? Since the distance a is positive, then the square root of the distance a to the square is equal to the distance a and, in addition, the square root of four is obviously equal to two. Given all that has been said, we finally have: »the height h is equal to the distance at Times Square root of three the whole divided by two« what had to be proved ».* In Middle School and high schools, as we can see, the teachers themselves encounter this problem and, as a result, they often speak with much confusion. For example: the use of the word "then" is frequently confused with the terms "implication"; Therefore; equality; equivalence; I take it, etc. ". According to our personal opinion, we would like to formally recommend to teachers-educators the following: to pronounce by their literal name any symbol or notation, even if we do not write it on the board or on the notebook, in order to raise awareness to the application the learners in their mathematical writing. The reason is simple. When reading an abbreviation or mathematical notation, many words are often swallowed, and learners may ignore them to the university itself, creating a source of failure in their training.

Let us take as an example: the height h is equal to plus or minus square root of three times the distance a squared, divided by four. When writing in mathematical terms, the sentence takes the form: $h = \pm \sqrt{\frac{3a^2}{4}}$. Often we, teachers, say in pronouncing the relation above: "h equals more or less square root of three squares out of four". It is good here to notice that the habit of abuse of language is to be avoided, especially if one faces the uninitiated to the matter more for the verb "to equal" it is early to use "to be equal". For example, the word "on" does not have exactly the same meaning as "divided by"; it is therefore recommended to get used to saying "divided by" instead of "on". Here we see that by going through "shortcuts", the appropriate terms for use are lost, so that learners might never think about it. And, of course, a false language taken without consciousness leads to a false, certain understanding, that is to say, the knowledge transmitted risks being erroneous. Finally, we say, and repeat, that introducing mathematical terms and using them in their exact meaning is not an easy task. It requires recognized professional pedagogical training and daily training.

VII- CONCLUSION

In the present work, we have seen that, in grammar and / or mathematical writing, the logical connectors are morphemes (adverbs, conjunctions of coordination or subordination, sometimes even interjections), which establish a connection between two statements, or even between a statement and an enunciation. They group logical connectors and argumentative connectors like "but", that is, words that, in addition to their connecting role, insert the connecting statements into an argumentative framework. The study of connectors thus integrates the perspectives of text grammar (concerned with the cohesion of the text) and that of pragmatics (interested in the argumentative orientation of statements and the relation of interlocution). Their role is to make a text or

proposals more fluid and better organize it. The absence of connector between two statements is in itself a form of junction. We can mention the following connectors:

Addition: and, moreover, then, besides, not only ... but still, moreover, as well as, equally, all in;

Alternative: or, either ... either, sometimes ... sometimes, or ... or, only ... but still, one... the other, one side ... the other, part ... of the other part;

But: so that, for that, lest, in view that, so that;

Cause: for, indeed, actually, as, because, expected, expected, seen, as given, thanks to, because of, consequence of, in respect of, because of, fact that, to the extent that under the pretext that, considering;

Comparison: like, similarly, similarly, similarly, similarly, similarly, the same, as, according as, as if;

Concession: despite, despite, though, although, while, whatever, even if, it is not that, certainly, of course, obviously, it is true that, however;

Conclusion: in conclusion, to conclude, by way of conclusion, in short, so, therefore, in short, in a word, consequently, finally, finally, finally;

Condition: Assumption: if, in the case, provided that, provided that, unless, assuming that, provided that, assuming that, assuming that, in the event that, in the case where, probably, no doubt, apparently;

Consequence: therefore, also, hence, "if Conditions, then Conclusion", thus, so therefore, consequently, so that, hence, accordingly, consequently, consequently, so that, so that, so that, so that, so and so that;

Classification, enumeration: firstly, firstly, firstly, first, first, second, second, second, second, second, second, third, then finally, to conclude, finally;

Explanation: to know, namely, that is, to be;

Illustration: for example, as thus, this is how it is the case of, inter alia, among others, in particular, in the image of, as illustrated, as underlines, such as;

Justification: because, that is, because, because, so that, thus, it is so that, not only ... but again, because of;

Addition: then, thus, also, moreover, in fact, in fact, moreover, in the same way, also, then, then, moreover, moreover, moreover, moreover;

Opposition: but, however, gold, on the other hand, while, however, against, while, nevertheless, on the contrary, for its part, on the other hand, despite, despite, notwithstanding, instead of, d on the one hand ... on the other hand;

Restriction: however, however, nevertheless, however, apart from,... except, apart from, except, failing, except, except, solely, simply;

Time: when, when, before, after, while, since, since, while, at the same time, while, at the time. All this has been recalled, corrected and presented because the editorial staff has a privileged and important place in teaching.

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