International Journal of Multidisciplinary and Current Educational Research (IJMCER)
ISSN: 2581-7027 ||Volume|| 1 ||Issue|| 4 ||Pages|| 15-27 ||2019||

Relevance and feasibility of early teaching of hyperbolic geometry according to Henri Poincaré's model

[1]Armand, [2]André Totohasina, [3]Daniel Rajaonasy FENO, [4]Ambeondahy

[1]*Lycee MixteAntsiranana Madagascar*

[2]*Department of Mathematics and Informatics Application University of Antsiranana Madagascar*

[3]*Department of Mathematics and Informatics Application University of Toamasina Madagascar*

[4] *Higher Institute of Science and Technology of Mahajanga, University of Mahajanga, Ex-LOLO Building Mahajanga-be Madagascar*

ABSTRACT **:** *In everyday life, there are several images architectures such that Euclidean geometry is not enough to build them, nor to optimize creativity and aesthetic education which are major issues in the teaching of geometry. However, the curriculum, almost worldwide, engender a limited perception tending to become indelible face noneuclidean geometric figures but frequent! This is a very persistent and even blocking pregnancy with Euclidean geometrical figures. Such learners who have become adults will risk missing out on these beautiful hyperbolic or spherical concave polygonal figures in engineering and artistic creativity. The present work makes explicit, in particular, the relevance of the teaching of at least one noneuclidian geometry since the middle school, as the real hyperbolic geometry from Poincaré model; then it will be continued and reinforced in high school terminals through analytical approach.*

KEY WORDS*: Stakes of geometries, Euclidean geometry, hyperbolic geometry.*

I. INTRODUCTION

On the epistemological level, since the nineteenth century, the history of mathematics attests the existence of other geometries besides the Euclidian, among others, the hyperbolic geometry of Lobachevski, geometry Riemann, etc. The mathematician Henri Poincaré was, between others, the pioneer of the modeling of analytic hyperbolic geometry [1], [8] [9] [10], [11]. With successive school programs, all high school and second, first and final science students in high school study geometry in its Euclidean form only [5], [6]. In general, according to the school programs that have followed one another, geometry must be taught exclusively to Euclidean flat or spatial geometry in middle and high school science students[4]. This type of school program would engender a limited perception that tends to become indelible in the face of noneuclidian geometric figures but frequent! This is a very persistent and even blocking pregnancy to Euclidean geometrical figures[2]. Such learners who have become adults may miss out on these beautiful hyperbolic or spherical concave polygonal figures in engineering and artistic creativity. However, in everyday life, there are several image architectures such that Euclidean geometry is not sufficient to build them, nor to optimize creativity and aesthetic education which are major issues in the teaching of geometries, and no longer "geometry", because it will now be appropriate to relativize or specify what geometry is it. Then the question arises about geometry:"How to create an opportunity for an unstable imbalance in the face of this lasting and almost exclusive impregnance to Euclidean geometry? ". To this end, this paper proposes, in particular, from the result of a diagnostic survey, with regard to the challenges of teaching-learning geometries, the relevance of teaching at least one nonEuclidean geometry from the middle school. Then he will be continued and reinforced in high school terminations.

II. HOMOGRAPHY AND NONEUCLIDEAN HYPERBOLIC GEOMETRY (POINCARÉ'S MODEL AND LOBACHEVSKI GEOMETRY)

2.1. Euclidean geometry

It is well known that the geometry in the plane seen by Euclid is based on the following five axioms (also called Euclid's postulates):

a) There is always a straight line that passes through two points of the plane;
b) Any segment can be extended along its direction in a straight line (infinite);
c) From a line segment, there is a circle whose center is one of the points of the segment and whose radius is the length of the segment;
d) All right angles are equal to each other;

e) Given a point and a line not passing through this point, there exists a single line passing through this point and parallel to the first one.

2.2. Hyperbolic geometry or geometry with several parallels

In mathematics, hyperbolic geometry (sometimes called Lobachevski geometry) is a noneuclidean geometry satisfying the first four postulates (Cf. 1.2. a), b), c) d)) of Euclidean geometry but not the fifth (Cf. e)). The goal of the quest for a model is to meet the following requirements:

a) Have a plan that can be represented as part of a Euclidean plan or space;
b) Have a plan that looks like a plan and, in any case, that is of dimension 2, so a surface;
c) Have lines that look like straight lines, or at least simple lines, if possible;
d) Maintain the usual incidence properties;
e) To have a compatibility between the invariants of the geometries (length, angle) and those of the model (by its Euclidean structure), thus to have an isometric and / or conformal representation, with, in particular, circles which resemble circles;
f) Finally, have a metric structure on the model so that the lines appear as geodesics for this structure (the shorter path lines)

In fact, this model search is totally hopeless if we do not reduce the requirements. In the case of elliptical geometry, one can only have a conformal model with arcs of circles instead of straight lines. Moreover, these models will not really be parts of a Euclidean space (we will always have to identify points). This is because the elliptical plane is a projective plane (a plane is not steerable so, it does not dive like a surface of $\mathbb{R}^3$). This time, the Euclidean postulate of parallels is replaced by the postulate that by a point outside a line passes more than a parallel line: it is shown that then (Figure 1), there is an infinity of lines parallel. In hyperbolic geometry, the theorem of Pythagoras is no longer valid and the sum of the inner angles of a triangle is no longer equal to 180°, more precisely, it is always less than this value. A straight line is always defined as the line of shorter path joining two points on a surface. Lobachevski, Klein and Poincaré have created models of nonEuclidean geometries in which we can trace an infinity of parallels to a given line and passing through the same point. In two dimensions, we can cite: the Poincaré disk, the Poincaré half-plane [9], [13], [14].

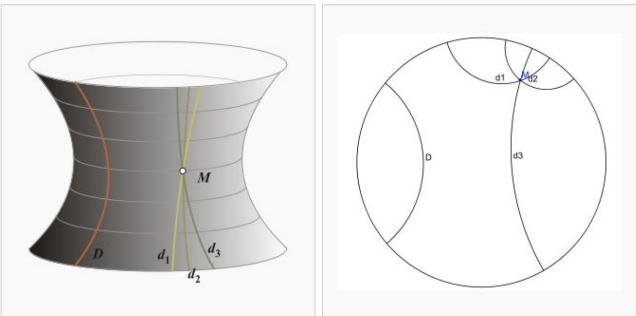


Figure 1 – Graphic representation of Lobachevski and Poincaré discs

There are indeed an infinity of rights which, liked_1, d_2, d_3, $d_4$go through the point *M* and are parallel to the line *D* (Figure 1).

2.3. Definition of the half-plane of Poincaré and hyperbolic straight lines

We consider the following situation, close to that of Poincaré's half-plane, which we shall see later, but simpler. We place ourselves in $\mathbb{R}^2$ and the new plane is the upper half-plane: $D_m = \{x + iy / y \geq 0\}$. The lines of$D_m$ are either half-straight lines parallel to the y-axis contained in D_m (which will be called the lords because they are less numerous), or the semicircles of origin a point of the x axis contained inD_m (the seignior). It is immediate that axioms of Euclid-Hilbert incidence and order are answered. The postulate of parallels, it is not. Indeed, ifD_m is a lord and at a point ofD_m, then there is a single parallel toD_m passing through A, while at the same time, if D_m is a manant, there is an infinity (all those who cutD_m in the lower half plane). This shows in passing that this geometry is not homogeneous on the straight side, which follows to disqualify it, considering the Euclidean geometry.

III. HOMOGRAPHYS RETAINING THE HALF-PLANE OF POINCARÉ

3.1. Definitions and properties

In the commutative field$\mathbb{R}$ of real numbers, a homographic function is a function of$\mathbb{R}$in itself defined by:

$h(z) = \frac{az+b}{cz+d}$, where *a*, *b*, *c* and *d*are elements of$\mathbb{R}$ and $ad \neq cd$. We forbid that*ad* - *bc*nonzero because the corresponding function would be constant(J.G. Semple & G.T. Kneebone. 1952), (O. Faugeras. 1992) so trivial. It happens that the condition "c not zero" is added, because the case *c = 0* would correspond to the affine functions; we then lose the group structure of the set of homographic functions provided with the application composition. We will then remember that a homographic function is a homeomorphism of: $\mathbb{C} - \left\{\frac{c}{d}\right\} \to \mathbb{C}$. In the following, we note by$\mathcal{H}$ the set of the homographys.

3.2. Geometric interpretation of complex homography$h_{a,b,c,d}(z) = \frac{az+b}{cz+d}$

It is clear that if $c = 0$ then h is the direct similarity but, if $c \neq 0$, so we can write $h_{a,b,c,d}(z) = \frac{a}{c} - \frac{ad-bc}{c^2} + \frac{1}{z+\frac{d}{c}}$(this is the decomposition into simple elements of the rational fraction of the homographic $\frac{az+b}{cz+d}$) which shows that $h_{a,b,c,d}$ is composed of an expression translation: $h_1(z) = z + \frac{a}{c}$, a similarity$h_2(z) = \left(\frac{ad-bc}{c^2}\right) z$, an inversion $h_3(z) = \frac{1}{\bar{z}}$of the center *O* and power 1and orthogonal axis symmetry of the real$h_4(z) = \bar{z} + \frac{d}{c}$.

Note 1. *For information, thanks to the functions associated with the inversion to return the infinite to a finite number, then the homography is well suited to exploit for the Video and Camera with the Zoom function.*

.

3.3. Homography keeping Poincaré's half-plane

We consider the whole$\mathcal{H}^+$homographys with coefficients *a, b, c, d* real verifying*ad* - *bc* = 1 and all$\mathcal{H}^-$anti-homography with real coefficients too, but checking this time*ad* - *bc* = -1 both are of simple hyperbolic group.

Let$\mathcal{H}^\pm = \mathcal{H}^+ \cup \mathcal{H}^-$.

Proposition 1.

(i) *The compound of two elements of*$\mathcal{H}^+$ *is in* $\mathcal{H}^+$.

(ii) *For every h element of* $\mathcal{H}$ *, we have h(* D_m *) =* D_m *where* D_m *is the half-plane defined by*$D_m = \{x + iy / y \geq 0\}$*called half-plane of Poincaré.*

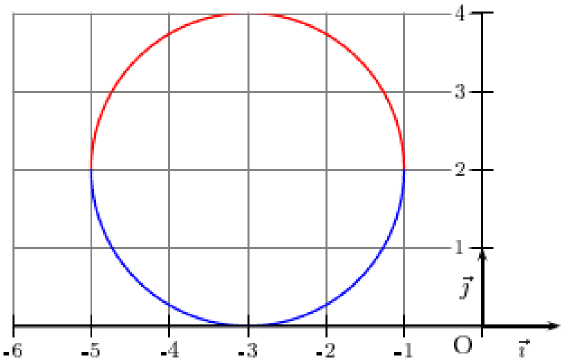
Subsequently, the homography are indeed the ideal geometric transformations to work according to the model of Poincaré.

IV. CONSTRUCTION OF HYPERBOLIC LINES

4.1. Reminder and definition

Definition 1. *The graphical representation of a circle of center I (a, b) and of radius r is nothing other than the union of two curves representative of two functions* $F^+_{(a,\ b,\ r)}$ *and* $F^-_{(a,\ b,\ r)}$ *defined by:* $F^+_{(a,\ b,\ r)}(x) = +\sqrt{r^2 - (x-a)^2} + b$ *and* $F^-_{(a,\ b,\ r)}(x) = -\sqrt{r^2 - (x-a)^2} + b$. *These two functions are called characteristic functions of this circle.*

Exemple 1.*The graphical representation of the circle C of center I (-3, 2) and of radius r = 4 is none other than the union of two curves representative of two functions* $F^+_{(-3,\ 2,\ 3)}$ *and* $F^-_{(-3,\ 2,\ 3)}$ *defined by:* $F^+_{(-3,\ 2,\ 3)}(x) = +\sqrt{3^2 - (x+3)^2} + 2$*and* $F^-_{(-3,\ 2,\ 2)}(x) = -\sqrt{3^2 - (x+3)^2} + 2$*(Cf. Figure 2).*



4.2. Hyperbolic lines

As we have already defined above that the hyperbolic straight lines are the following parts of$D_m$whose:

- the vertical half-lines of origin located on the x-axis;

That is the set notedD_a defined by: $D_a = \{(a,\ y) \in \mathbb{R}^2 /$ a: constant and $y \geq 0\}$ and

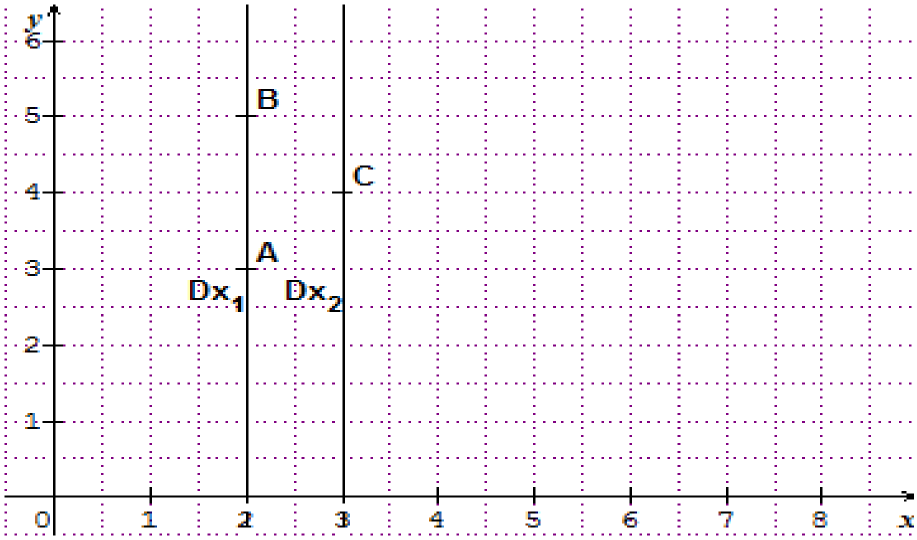
- the semicircles on the upper plane centered on the abscissa axis.

That is the whole noted$D_{(a,\ r)}$defined by:

$D_{(a,\ r)} = \{(x,\ y) \in \mathbb{R}^2 / \quad (x-a)^2 + y^2 = r^2 \ and \quad y \geq 0\}$

In the rest of the text, all the figures are obtained under Sine qua non.

Example 2. *The set*$D_2 = \{(2,\ y) \in \mathbb{R}^2 / \ y \geq 0\}$*and the set* $D_3 = \{(3,\ y) \in \mathbb{R}^2 / \ y \geq 0\}$*are represented on the(figure 3).*



4.3. Characteristic function determining the Poincaré's semicircle

Definition 2.*We consider a semicircle of center I (a, 0) and radius r centered on the abscissa axis or simply the whole noted $D_{(a,\ r)}$ defined by:*

$D_{(a,\ r)} = \{(x,\ y) \in \mathbb{R}^2 / \quad (x-a)^2 + y^2 = r^2 \ et \quad y \geq 0\}$.

We call the characteristic function of the semicircle defining the hyperbolic line of Poincaré the continuous function and defined on the bounded interval$[a - r\ ;\ \ a + r]$which can trace the semicircle directly and has for expression: $F_{(a,\ r)}(x) = \sqrt{r^2 - (x-a)^2}$.This is a special case of the function$F^+_{(a,\ b,\ r)}$taking$b$ = 0.

Example 3*The set*$D_{(-2,\ 3)} = \{(x,\ y) \in \mathbb{R}^2 / \quad (x+2)^2 + y^2 = 3^2 \ and \quad y \geq 0\}$*is shown in Figure 4.*$D_{(-2,\ 3)}$*is the graphical representation of the function*$F_{(-2,\ 3)}$*defined by:* $F_{(-2,\ 3)}(x) = \sqrt{3^2 - (x+2)^2}$.

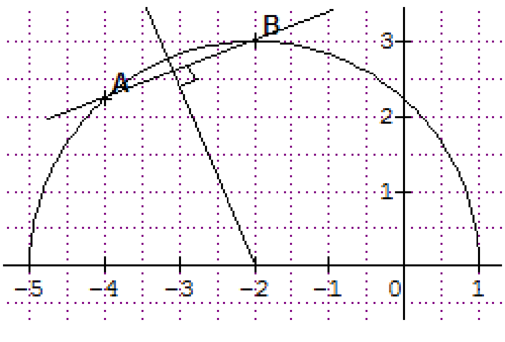


Figure 4 – A semicircle centered on the x-axis

The two lines (vertical and semicircle) each have two points so that there is only one straight line that passes at these points. This proves that Euclidean geometry is included in hyperbolic geometry. It could be the limit of hyperbolic geometry.

4.4. Construction of a triangle in a half-plane of Poincaré

To build this triangle, first draw three semicircles in this half-plane.

Example 4.*Are four characteristic functions* $F_{(-5,\ 3)}$, $F_{(0,\ 3)}$, $F_{(2,\ 3)}$, $F_{(4,\ 3)}$ *defined by :*

$F_{(-5,\ 3)}(x) = \sqrt{3^2 - (x+5)^2}$, $F_{(0,\ 3)}(x) = \sqrt{3^2 - (x)^2}$, $F_{(2,\ 3)}(x) = \sqrt{3^2 - (x-2)^2}$, $F_{(4,\ 3)}(x) = \sqrt{3^2 - (x-4)^2}$.The graphical representation of each of these four functions is the four semicircles below (Figure 5) and points *A*, *B*, *C* are such that$A(1,\ \sqrt{8})$, $B(2,\ \sqrt{5})$, $C(3,\ \sqrt{8})$. We see that *ABC* (figure 5) constitutes a triangle, such that the sum of the three angles, which constitute this triangle, is less than 180^o.

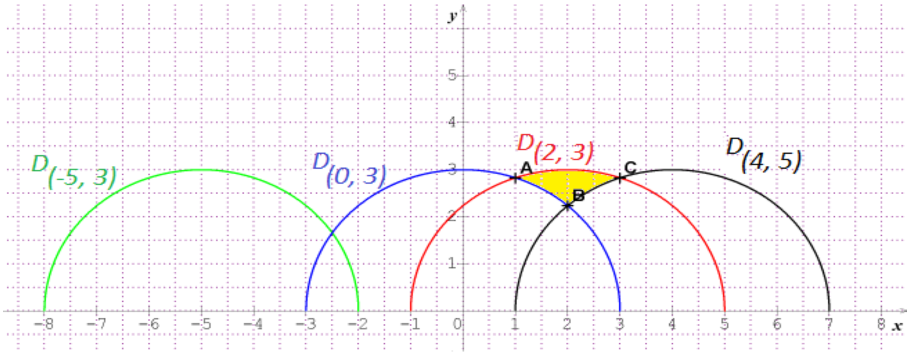


Figure 5 – Semicircles centered on the x-axis

In addition, the two lines $D_{(2,\ 3)}$ and $D_{(4,\ 5)}$are parallel to the line$D_{(-5,\ 3)}$. Or, point *C* is a point outside the line $D_{(-5,\ 3)}$, but the two lines $D_{(2,\ 3)}$ and $D_{(4,\ 5)}$go through this point. This shows us that the 5thEuclid's postulate is no longer valid.

Note 1. *The possibility of constructing other hyperbolic polygons is deduced from that of hyperbolic triangles*.

Example 5.*Consider the following five feature features* $F_{(-3,\ 3)}$, $F_{(0,\ 3)}$, $F_{(1,\ 3)}$, $F_{(1/2,\ 1)}$, $F_{(4,\ 3)}$ *defined by:* $F_{(-3,\ 3)}(x) = \sqrt{3^2 - (x+3)^2}$, $F_{(0,\ 3)}(x) = \sqrt{3^2 - (x)^2}$, $F_{(1/2,\ 3)}(x) = \sqrt{3^2 - (x-1/2)^2}$, $F_{(4,\ 3)}(x) = \sqrt{3^2 - (x-4)^2}$.*They are shown in Figure 6 and the points A, B, C, D, E are such that* $A(-1,\ \sqrt{5})$, $B\left(-\frac{3}{28},\ \frac{3\sqrt{55}}{28}\right)$, $C\left(-\frac{31}{28},\ \frac{3\sqrt{55}}{28}\right)$ $D(2,\ \sqrt{5})$ *and* $E\left(\frac{1}{2},\ \frac{\sqrt{35}}{2}\right)$, $G\left(\frac{15}{16},\ \frac{3\sqrt{3}}{2}\right)$.

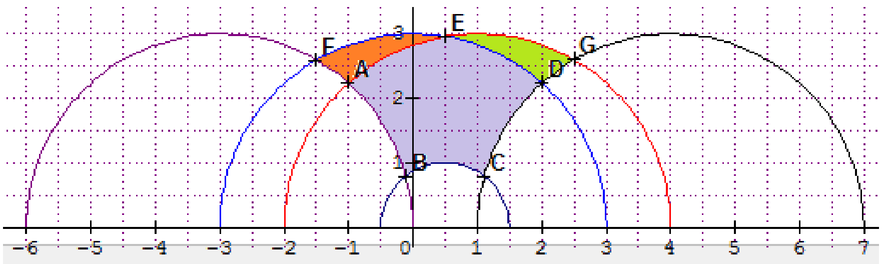


Figure 6 – The hyperbolic pentagon *ABCDE* and the hyperbolic triangles *AEF* and *DGE*

Facing the expression of the function $F_{(a,\ r)}$ such as $F_{(a,\ r)}(x) = \sqrt{r^2 - (x-a)^2}$ representing the upper semicircle curve of equation circle$D_{(a,\ r)} = \{(x,\ y) \in \mathbb{R}^2 (x-a)^2 + y^2 = r^2\}$we can draw a theorem that follows.

Theorem 4.1.*For all points A and B such as* $A(x_1, y_1)$ *and* $B(x_2, y_2)$, *any half-circle checking the expression*$F_{(a,\ r)}(x) = \sqrt{r^2 - (x-a)^2}$*passing in these points where*$x_1 \neq x_2$has for *a*center defined by

$$a = \frac{y_1^2 - y_2^2 - x_2^2 + x_1^2}{2(x_1 - x_2)}$$

and the radius$r = \sqrt{y_1^2 - (x_1 - a)^2}$ where $r = \sqrt{y_2^2 - (x_2 - a)^2}$.

Prove

Let *A* and *B* be such$A(x_1, y_1)$and $B(x_2, y_2)$, we have:

$$\begin{cases} A \in F_{(a,\ r)} \Leftrightarrow & y_1^2 = r^2 - (x_1 - a)^2 \\ B \in F_{(a,\ r)} \Leftrightarrow & y_2^2 = r^2 - (x_2 - a)^2 \end{cases},$$

Thus, $\begin{cases} y_1^2 = r^2 - (x_1 - a)^2 & (1) \\ y_2^2 = r^2 - (x_2 - a)^2 & (2) \end{cases}$

and (1)-(2) give $y_1^2 - y_2^2 = (x_2 - a)^2 - (x_1 - a)^2 \Leftrightarrow a = \frac{y_1^2 - y_2^2 - x_2^2 + x_1^2}{2(x_1 - x_2)}$.

Hence the stated theorem.

Corollary 2. *In the case where* $x_1 = x_2$*we obtain*$D_x = \{(a, \ y) \in \mathbb{R}^2 / a \ constant \ and \ y \geq 0\}$ *where* D_x*goes through the two points and*$a = x_1 = x_2$.

4.5. Homography and hyperbolic geometry

As we have already said in paragraph 3 subsection 3.3, any homography of $\mathcal{H}^+$keep the half-plane of Poincaré. So let's consider a homography h defined by $h(z) = \frac{z+2}{z+3}$and take advantage of the property above to find$h(ABC)$of the triangle defined in Figure 5.It is then immediate that$h(ABC) = A'B'C'$a triangle located on the same plane.

4.6. Finding coordinates of points$A'B'C'$ image de ABC par h

Recall in passing that for the visualization of hyperbolic figures in the half-plane of Poincaré, it is advantageous to combine and/or combine the use of GeoGebra software, Derive5, Sine qua non by exploiting their complementarity.

Indeed, $h(A) = A'$which is such that $A'\left(\frac{5}{6}, \ \frac{\sqrt{12}}{12}\right)$, $B'\left(\frac{5}{6}, \ \frac{\sqrt{5}}{30}\right)$, $C'\left(\frac{19}{22}, \ \frac{\sqrt{2}}{22}\right)$. Hence the triangle imageABCsuch as$A(1, \ \sqrt{8})$, $B(2, \ \sqrt{5})$, $C(3, \ \sqrt{8})$by homography h is the triangle$A'B'C'$such as$A'\left(\frac{5}{6}, \ \frac{\sqrt{12}}{12}\right)$, $B'\left(\frac{5}{6}, \ \frac{\sqrt{5}}{30}\right)$, $C'\left(\frac{19}{22}, \ \frac{\sqrt{2}}{22}\right)$are all located on the same plane of Poincaré (Figure 7).

4.7. Triangle construction$A'B'C'$

We begin by first finding the characteristic functions of semicircles passing respectively by the points$A'$and B', A' and C', B' and C'. The point abscissaA' and B', are equal, therefore the semicircle image passing through the points *A* and *B* is the half-line passing throughA' and B', on the upper axis of equation$x = \frac{5}{6}$and the semicircle image passing through *A* and *C* is the semicircle passing through the pointsA' and C'having the characteristic function $F_{A'C'\left(\frac{11}{12}, \ \frac{3}{16}\right)}(x) = \sqrt{\left(\frac{3}{16}\right)^2 - \left(x - \frac{11}{12}\right)^2}$ and the semicircle image passing through *B* and *C* is the semicircle passing through the points B' and C' having the characteristic function$F_{B'C'\left(\frac{33}{40}, \ \frac{3}{40}\right)}(x) = \sqrt{\left(\frac{3}{40}\right)^2 - \left(x - \frac{33}{40}\right)^2}$. Hence the triangle$A'B'C'$is built on the figure7.

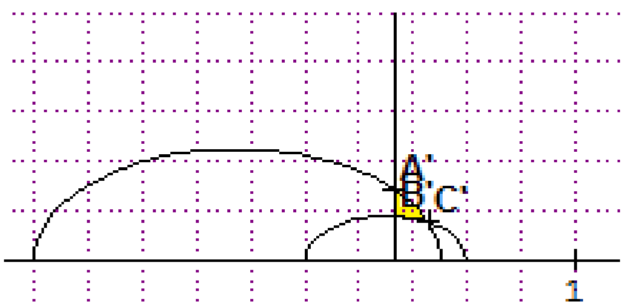


Figure 7 – The triangle $A'B'C'$image of triangle *ABC* by homography*h*

The situation is quite embarrassing because$A'B'C'$is almost illegible here for lack of scale. This result leads us to use homographyh^{-1} définie par: $h^{-1}(z) = \frac{-3z+2}{z-1}$assuming that$h^{-1}$has the same determinant as *h*. Nevertheless, it is possible to overcome this problem as indicated below.

4.8. Finding coordinates of points $A''B''C''$, image of*A*, *B*, *C* par h^{-1}

Here we have: $h^{-1}(A)=A''$which is such that: $A''\left(-3,\ \frac{\sqrt{2}}{4}\right)$, $B'\left(-\frac{19}{6},\ \frac{\sqrt{5}}{6}\right)$, $C'\left(-\frac{19}{6},\ \frac{\sqrt{2}}{6}\right)$. Hence the three points$A''B''C''$, which constitute a triangle$A''B''C''$, image of triangle *ABC* by$h^{-1}$are all located on the same plane of Poincaré (figure 8) such as$A''\left(-3,\ \frac{\sqrt{2}}{4}\right)$, $B'\left(-\frac{19}{6},\ \frac{\sqrt{5}}{6}\right)$, $C'\left(-\frac{19}{6},\ \frac{\sqrt{2}}{6}\right)$.

4.9. Triangle construction$A''B''C''$

We begin by first finding the characteristic functions of semicircles passing respectively by the points$A''$and B'', A'' and C'', B'' and C''.The points abscissaA' and B', are equal, therefore the semicircle image passing through the points *A* and *B* is the half-line passing through A'' and B'', on the upper axis of equation$x=-\frac{19}{6}$and the semicircle image passing through *A* and C is the semicircle passing through the pointsA'' and C''having the characteristic function$F_{A''C''\left(-\frac{25}{8},\ \frac{3}{8}\right)}(x)=\sqrt{\left(\frac{3}{8}\right)^2-\left(x+\frac{25}{8}\right)^2}$and the semicircle image passing through *B* and *C* is the semicircle passing through the points B' and C' having the characteristic function$F_{B''C''\left(-\frac{23}{8},\ \frac{3}{8}\right)}(x)=\sqrt{\left(\frac{3}{8}\right)^2-\left(x+\frac{23}{8}\right)^2}$. Hence the triangle$A''B''C''$is constructed in Figure 8.

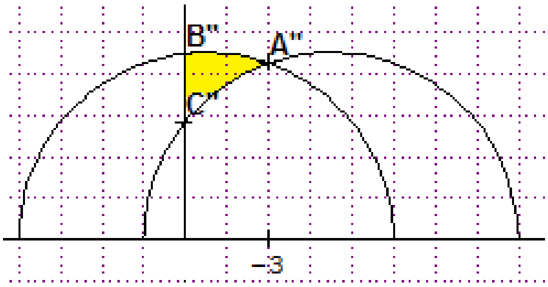


Figure 8 – The triangle $A''B''C''$image of triangle ABC by homographyh^{-1}

Indeed, by comparing the results obtained with homography h and its inverse which is of the same type as this one, we see that the image ofh^{-1} (figure 8) is more readable than the one given by *h*.

V. CHALLENGES OF TEACHING-LEARNING GEOMETRIES

The student learns only by acting and therefore by being involved in the learning teaching scenario. Contrary to the behaviorist conception, acquiring knowledge is not a process of stacking, but rather the transition from a phase of equilibrium to a new equilibrium through a phase of socio-cognitive conflict imbalance caused by a contradiction between pre-conceptions and the situation we are facing. Etymologically, the term "geometry" comes from the two roots "geo" and "metry" meaning respectively "earth (gaia in Greek)" and "measure (metron in Greek)". After these historical references on pedagogy and on the geometry and stakes of its teaching-learning at school levels, we present in a very simplified and quite intuitive way the analytical hyperbolic geometry related to complex homographys. Then, we will try to ascertain the relevance and the feasibility of the early teaching of this noneuclidean geometry according to the Poincaré model: this reform concerns the middle and high school level [3].

5.1. Experimental study

As a guide, here is a question about the recognition and naming of the figures of Euclidean geometry and hyperbolic geometry that we asked 40 students and 10 teachers' researchers of mathematics and physical sciences (Cf. Figure 9)

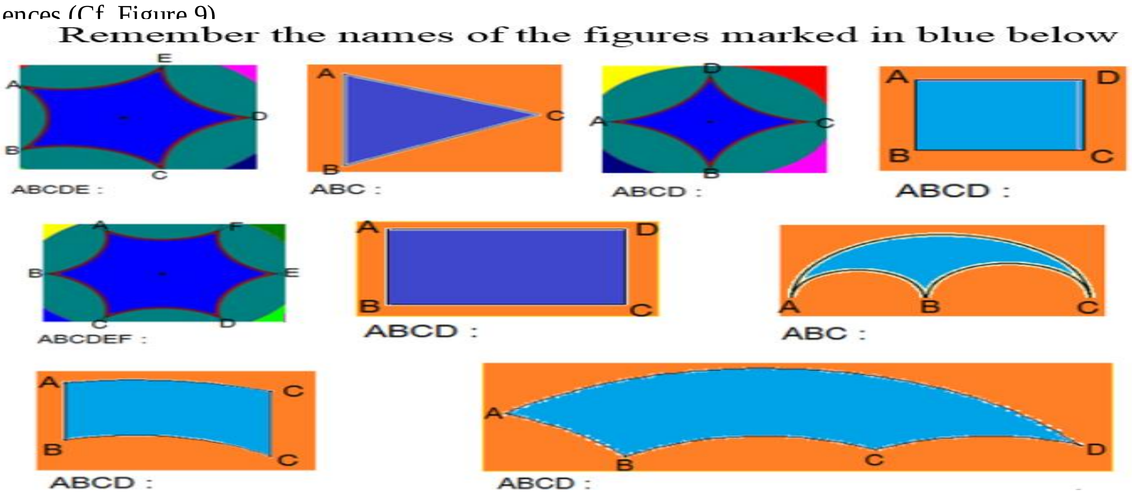


Figure 9 - Test Topic

The survey gave the result represented by Figure 10.

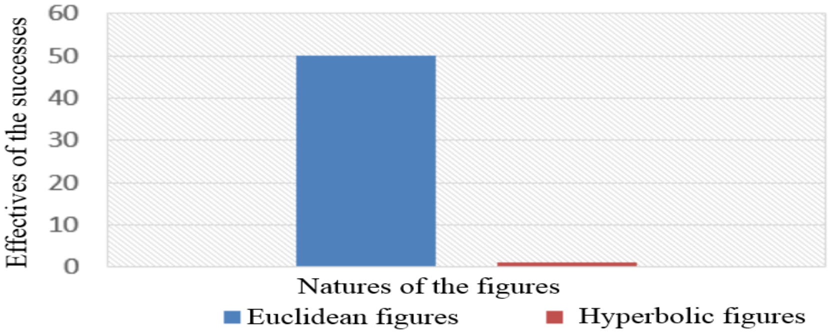


Figure 10 – Result of the investigation

At the end of this test, we noticed that nobody can correctly name the figures relating to hyperbolic geometrics, most of them have no words to recognize them. The interpretation of this fact is obvious, given the curricula followed. Here are some sample answers regarding the current test (Cf. Figure 11 and 12) :

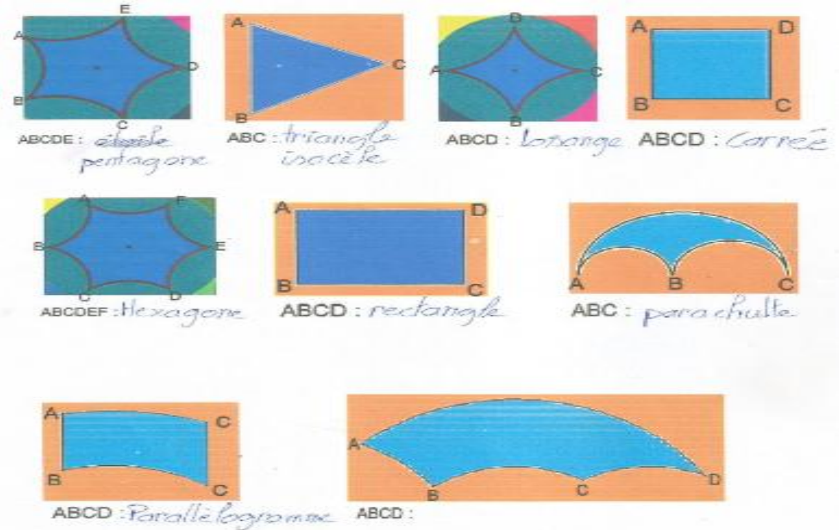


Figure 11 - Test Copy

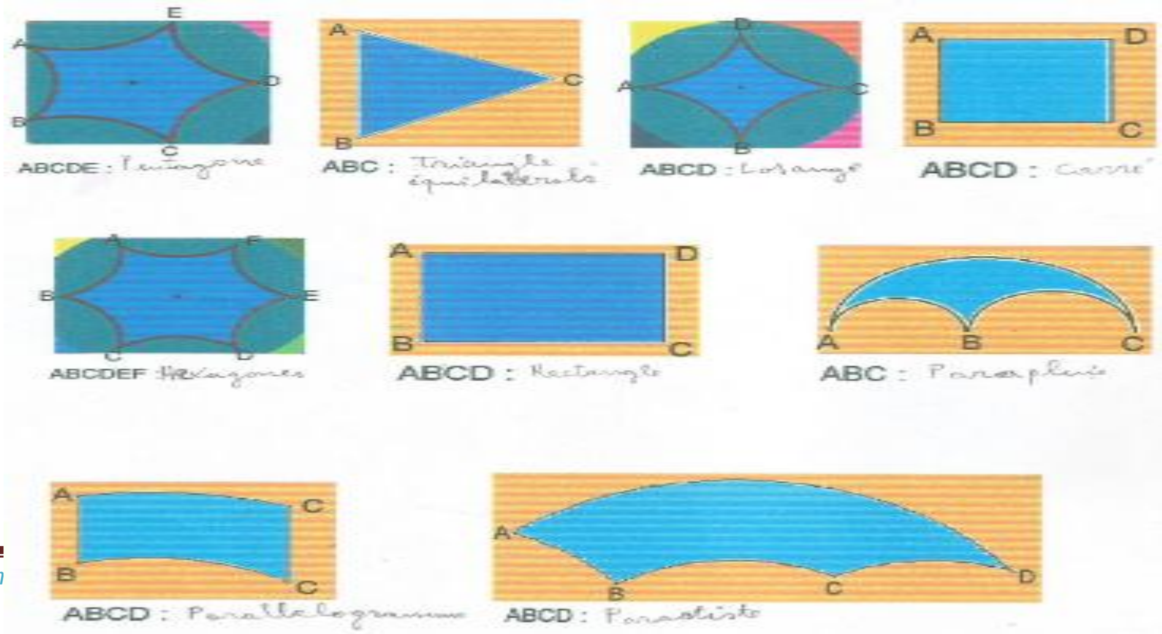


Figure 12 - Test Copy

5.2. Analysis of the survey result

Presumably, the non-learning of hyperbolic geometry and the pregnance to the Euclidean geometry could be the reason for this failure. But, in everyday life, he There are several image architectures such that Euclidean geometry is not sufficient for build them, nor to optimize the creativity and aesthetic education that are issues majors of geometry education: as an indication, let's appreciate the polygons described in the annexure (note that all these polygons are extracted from Regular hyperbolic polygons and its translated by Marcel Morales). All this shows, partially, the relevance of teaching at least one noneuclidean geometry from the middle school; then he will be continued and reinforced in high school terminals. For example, here are the three founders of the geometries that exist in the literature (Cf. Figure 13).

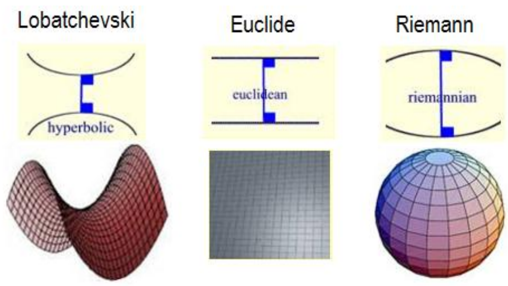


Figure 13 – Three geometries existing in the literature

VI. RESEARCH AND METHOD IN DIDACTICS

The didactics of mathematics is commonly perceived as the in-depth reflection on the teaching-learning process of precise mathematical concepts, on the process of their diffusion, and on the origins and remedies of the eventual difficulty and learning obstacles of these mathematical concepts. To undertake research in didactics of mathematics, it is permissible to function as follows:

a- Propose tasks to a number n ($n \geq 50$) of pupils/teachers whose realization can implement several definitions of the concepts in question;

b- Interpret student / teacher productions to see if certain definitions and / or apprehensions were favored by the target population. It is also, through confrontations of methods deployed by students, to promote learning by some students of the properties of the concept through its use.

VII. COLLEGE LEVEL

7.1. Child from 11 to 12 years old: 6th class

They must be able to deepen, to develop the knowledge acquired on the rights to the Primary school and to use properties and definitions to build and / or to justify. That is, he must be able to (of):

- name a line (AB), (xy), (D);
- recognize and place aligned points, nonaligned points;
- draw the line passing for two given distinct points;
- recognize and draw lines intersecting;
- know and use the notation (D) ⊥ (L), (D) // (L);
- recognize on a coded figure two perpendicular lines, two parallel lines;
- build using the ruler and the square:
  - ✓ the line perpendicular to a given line and passing through a given point;
  - ✓ the line parallel to a given line and passing through a given point;
  - ✓ heck that two straight lines are perpendicular;
  - ✓ justify that two lines are parallel or that they are perpendicular using definitions and properties.

Morality 1.*For the student having acquired this knowledge above, one can indeed introduce in this class the* *notion of Poincaré half-plane followed by the construction of hyperbolic straight lines.*

7.2. Child from 12 to 13 years old: 5th class

Primarily, the student in this category must be able to use the notion of distance to characterize a segment and a triangle, he must also be able to have a first notion of planarization. That is, he must know some characteristic properties of particular triangles, recognize and build particular triangles.

Morality 2. *For the student who has acquired this knowledge above, we can indeed introduce the hyperbolic geometry in this class but we limit ourselves to the notion of drawing a semicircle centered on the x axis passing through two points on the half-plane and the construction of all geometric elements, triangle, parallelogram etc. knowing that the sum of the angles of the triangle is less than* 180^o.

7.3. Child from 13 to 14 years old: 4th class

The general goal that he must be able to use orthogonal and central symmetries to justify configuration properties and a simple construction program. That is, he must be able to know the property of the right of the backgrounds and properties of particular lines of a triangle. More precisely, he must be able to (of)

- ➢ state the definition of the right of the midpoints;
- ➢ state the properties of the segment joining the midpoints of two sides of a triangle:
  - ✓ segment length
  - ✓ support direction of this segment;
  - ✓ use the direct property to justify that two lines are parallel;
  - ✓ use the reciprocal property to justify that a point is the middle of a segment;
- ➢ build the orthocenter of any triangle including:
  - ✓ One of the angles is obtuse;
  - ✓ the three angles are acute;
  - ✓ recognize the orthocenter of a right triangle (this is the top of the right angle);
  - ✓ construct the circle inscribed in a given triangle;
  - ✓ build the center of gravity of a triangle;
  - ✓ determine the center of gravity position.

Morality 3.*For the student who has acquired this knowledge above, one can indeed introduce the hyperbolic geometry in this class but one limits oneself until the introduction of notion of the inversion thus, it is made according to that He already masters the basic techniques for the study of vector computations and he also knows how to implement the elementary techniques for the vectorial study of the situations encountered in geometry. In a nutshell, the teaching of noneuclidean hyperbolic geometry at the college level is limited to the sketch of the construction of fundamental and elementary hyperbolic geometrical figures without entering into analytical studies.*

7.4. Child from 14 to 15 years old: 3rd class

The pupil of this class must mainly be able to know the direct and reciprocal properties of Thales, and use these properties to justify and build. That is, he must know the direct and reciprocal properties of Thales and use these properties to justify and build. He must also be able to master and complete the knowledge acquired in previous classes on symmetries and translations and to use, in a more efficient way, symmetries and translations to demonstrate and to build and still to acquire a first notion on the composition of the two transformations. In other words, he must be able to

- ➢ recognize a configuration of Thales in a triangle;
- ➢ correctly state the direct and reciprocal properties of Thales;
- ➢ Thales properties for:
  - ✓ share a segment in a given report;
  - ✓ build the fourth proportional of the numbers a, b, and c taken in that order;
  - ✓ calculate the distances;
  - ✓ justify a parallelism of the lines.
- ➢ build a triangle similar to another given;
- ➢ recognize similar triangles on a given configuration;
- ➢ justify that two given triangles are similar;
- ➢ solve simple problems involving similar triangles;
- ➢ recognize a configuration of Thales in the general case
- ➢ know some simple applications of Thales theorem in the general case.
- ➢ recognize symmetries and translations;

- use symmetries and translations to justify:
  - ✓ a property of a configuration;
  - ✓ a construction program;
  - ✓ to construct the image of a point, of a simple figure by:
  - ✓ the compound of two orthogonal symmetries of parallel axes, of perpendicular axes
  - ✓ the compound of two central symmetries;
  - ✓ the compound of two translations.
- justify that:
  - ✓ the compound of two orthogonal symmetries of parallel axes is a translation;
  - ✓ the compound of two orthogonal symmetries of perpendicular axes is a central symmetry;
  - ✓ the compound of two central symmetries is a translation;
  - ✓ the compound of two translations is a translation.

Morality 4. *For the student who has acquired this knowledge above, one can indeed introduce the hyperbolic geometry in this class. But, we limit ourselves to the proof that homography is an ideal geometrical transformation to work according to Poincaré's model.*

VIII. HIGH SCHOOL LEVEL

8.1. Student aged 15 to 16 old: second class

The student must be able to use the tools put in place to solve problems concerning configurations. It must also be able to use the scalar product to demonstrate properties, to calculate vector standards, distances and angles, to solve orthogonality problems, to determine some metric relationships in the right triangle and then in any triangle. , to perform calculations in analytic geometry and to solve problems concerning the relative position of a line and a circle and that of two circles. Specifically, he must be able to

- perform distance and angle calculations using scalar product and demonstrated metric relationships;
- demonstrate geometric properties using the scalar product and demonstrated metric relationships;
- use the scalar product for:
- solve orthogonality problems;
- perform calculations in analytical geometry;
- check that a point belongs to a circle;
- find the equation of a circle knowing:
- its center and its radius;
- Find the center and radius of a circle knowing its equation.

Morality 5. *For the student who has acquired this knowledge above, we can indeed introduce hyperbolic geometry in this class. But one limits oneself to the exploitation of the inverse function to introduce notions of the real inversion and some properties of the inversion of center O and of power k and its applications in hyperbolic geometry.*

8.2. Pupil aged 16 to 17: first class

The student must be able to:

- define and construct centroids of 2, 3, 4 points in the plane;
- to look for geometric places using the centroid and the dot product: for the study of the transformation of the plane:
- to study a composite of homotheties and translation as well as homothety and rotation.

He must know the notion of plane similarity and its first properties, and he must also know how to use some of these transformations to solve simple problems.

- Plane Geometry (Plan Vector and Analytic Geometry)

The student must be able to define and build the centroid of 2, 3, 4 points of the plane and he must be able to find geometric places using the centroid and the dot product. He must also be able to perform analytical calculations on the right and on the circle;

- Transformation of the plan

The student must be able to compose transformations of the plan and he must also know some use of the transformations of the plan. He must also be able to acquire a first notion about isometries and similarities.

Morality 6. *For the student who has acquired all the above knowledge, we can indeed introduce the hyperbolic geometry in this class but it is limited to the test that homography is an ideal geometric transformation to work according to the model of Poincaré.*

8.3. Student 17 to 18 years old: terminal class

Any student in scientific terminology is normally already taught of such plane transformations, translation, rotation, orthogonal symmetry and homothety, more precisely the direct and indirect similarities conform to the following general objectives. He must be able to know and use certain properties of the center of gravity of n weighted points and determine coordinates of the center of gravity. He must also know how to use the center of gravity in solving geometry problems. He must also be able to systematically study translations, rotations and orthogonal symmetries in order to classify these isometries and he must know how to solve geometry problems by using these transformations.

8.4. Affine applications

The student must be able to:

- know what is an affine application and its few properties and study on sets, affine applications of the plan;
- solve problems using the analytical expressions of an affine application.

8.5. Flat geometry Isometry affine

The student must be able to:

- systematically study orthogonal translations, rotations and symmetries in order to classify these isometries;
- solve geometry problems using these transformations.

8.6. Flat similarities

The student must be able to:

- know and use the plane similarities and make the link between complex numbers and similarities.

8.7. Hyperbolic geometry according to the Poincaré model

The student must be able to:

- introduce complex homography and the complex expression of inversion;
- to deepen the power of homography on geometric transformation according to the Poincaré model; by introducing the characteristic function determining the Poincaré semicircle + Its use on the construction of the image of a semicircle passing by two points by means of three software indicated in subsection 4.6. In a nutshell, the teaching of noneuclidean hyperbolic geometry at the high school level can already open up to analytic studies.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

IX. CONCLUSION ET PERSPECTIVES

To conclude, this modest work has shown us that it is quite possible to introduce the teaching of noneuclidean geometry according to Henri Poincaré's model. But, we will limit ourselves to the sketch of construction of basic fundamental hyperbolic geometric figures using geometric construction materials (compass, ruler, etc.). The student learns only by acting and therefore by being involved in the learning teaching scenario. Contrary to the behaviorist conception, acquiring knowledge is not a process of stacking, but rather the transition from a phase of equilibrium to a new equilibrium through a phase of socio-cognitive conflict imbalance caused by a contradiction. Between preconceptions and the situation we are facing. Indeed, no one is unaware that the pedagogy of exclusively Euclidean geometry from infancy from 6 years old to adolescence has certainly given rise to a preconception of only perceiving Euclidean geometrical forms, a preconception that may be difficult to erase. , even indelible, has become an epistemological obstacle, but also a didactic obstacle to learning noneuclidean geometry. Moreover, we have shown the possibility of creating opportunities for an unstable imbalance in the face of this lasting and almost exclusive impregnance to Euclidean geometry. Of course, this could indeed anticipate a reform of the school curriculum aiming at the early introduction of plane hyperbolic geometry through complex homographic transformations, with a concern for social and cultural justice in the education of future generations. Thus, would it not be opportune, in this era of new technologies, to launch a reform proving to be relevant and quite feasible on school mathematics consisting in introducing at the college level the notion of plane hyperbolic geometry through visualization and construction elementary hyperbolic figures first, then analytically by investing complex homographys and anti-homographys in a half-plane of Poincaré in high school final, as reinforcement?

The authors declare that there are no conflicts of interest regarding the publication of this paper.

REFERENCES

[1] Armand, André Totohasina. Pertinence etfaisabilitéde l'introductionfonctionnellede l'ellipsedès la classe de première. In revue Didaktika. Formation et enseignement/apprentissage :stratégie, méthode, outils, pratiques. Pages 120-136. Volume N° 3 2019. Récupéré du site http://madarevues.recherches.gov.mg, 2019.

[2] Armand, André Totohasina. Uneréflexiontirée du vécu sur l'enseignement des transformations ponctuellesenterminale S. In revue Didaktika. Formation etenseignement/apprentissage: stratégie, méthode, outils, pratiques. Pages 137-148. Volume N° 3 2019. Récupéré du site http://madarevues.recherches.gov.mg, 2019.

[3] André Totohasina and Armand. Pertinence, faisabilitéd'enseignementprécoce de la géométriehyperbolique. In SaarbruckenAllemagne, editor, La maisond'éditionsuniversitaireseuropéennesISBN : 17978-3-330-87497-8, 2019.

[4] HerinainaElysé RAJAONARIMANANA, André Totohasina. Initiation aux TIC par Geogebradans des classes de mathématiques. In revue Didaktika. Formation et enseignement/apprentissage :stratégie, méthode, outils, pratiques. Pages 76-96. Volume N° 3 2019. Récupéré du site http://madarevues.recherches.gov.mg, 2019.

[5] S. Benhimane and E. Malis. Real-time image-based tracking of planes using efficient second-order minimization. In: IEEE/RSJ International Conference on Intelligent Robots Systems, Sendai, Japan, October, pages 176–186, 2004.

[6] F. Chaumetteand E. Malis and S. Boudet. 2 1/2 d visual servoing. In: IEEE Transaction on Robotics and Automation, pages 234–246, 1999.

[7] O. Faugeras. (1992). What can be seen in three dimensions with an uncalibrated stereo rig ? In Proc European Conference on Computer Vision, Santa Margherita Ligure, Italie, pages 563–578, 1988.

[8] O. Faugeras and F. Lustman. Motion and structure from motion in a piecewise planar environment. In: International Journal of Pattern Recognition and Artificial Intelligence, pages 485–508, 2012, 1988.

[9] YvoJacquier. La naissance de la géometrie: La géometrie avec les yeux des egyptiens. In : REPERES–IREM. No 87 avril, number 1, pages 2–22, 2012.

[10] E. Malis. Hybrid vision-based robot control robust to large calibration errors on both intrinsic and extrinsic camera parameters. In: European Control Conference Porto, PortugaSeptemberl, pages 2898–2903, 2001.

[11] E. Malis and F. Chaumette. Theoretical improvements in the stability analysis of a new class of model free visual servoing methods. In: IEEE Transaction on Robotics and Automation April, pages 176–186, 2002.

[12] M. Moakher. Means and averaging in the group of rotations. In: SIAM Journal on Matrix Analysis and Applications, number 1, pages 1–16, 2002.

[13] Jaen LusShabert. Les géométries non euclidiennes. In: REPERES - IREM. No01octobre, number 1, pages 69–91, 1990.

[14] Jaen LusShabert. Les géométries non euclidiennes. In: REPERES - IREM.No5 octobre, number 1, pages 69–91, 1991.

[15] J.G. Semple and G.T. Kneebone. Algebraic projective geometry. Oxford Science Publication, 1952.

[16] W.E. Dixon Y. Fang, D.M. Dawson and M.S. de Queiroz. Homography-based visual servoing of wheeled mobile robots. In: Proc. IEEE Conf. Decision and Control, pp, Las Vegas, NV, Dec, pages 2866–282871, 2002.

Annex

Fundamentals hyperbolic geometrics figures