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Mathematical method to convert certain functions via their respective compositions

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Smmury: *We propose in thispaper the response of the lessknown compositions of the few functionsnamely the composition of an affine function by a parabola function, a parabola by an ellipse, a line by an ellipse, etc. Among the variouspossibilities, in this article, wewouldbeinterested in its applications in physicsnotably on the study of rectilinear motion uniform and circular as for the conversion of their respective trajectories.*

KEY WORDS: *affine function, parabolic function, elliptic function, composition.*

I. INTRODUCTİON

Mathematics is often perceived as a theoretical science, useless in everyday life [16], [10], [19]and [12]. They are an irreplaceable tool for rigorous training and reasoning [14], [5], [13]and [17]. They prove to be an excellent tool for developing human intelligence by its crystallized component[9], [8], [18], [7]. Moreover, the teaching-learning of mathematics shows the possibility of concretizing objectives of the applications of the composition of functions,[6], [15] and [3]. Also, this paper concerns the response of the less known compositions of some functions namely by the composition of an affine function by a parabola function, a parabola by an ellipse, a line by an ellipse, etc.. Among the various possibilities, in this article, we are interested in its applications in physics notably on the study of rectilinear motion uniform and circular.In what follows, our work is divided into five sections. Section 2 gives some reminders and definitions. Section 3 discusses a technique for building an ellipse. Section 4 concerns the notion of compound functions. Section 5 introduces the less known composition of some functions. Section 6 gives a conclusion.

II. REMINDERS AND DEFINITIONS

In this section, we recall some definitions of an ellipse of lines and a parabola and sometimes followed by an example.

2.1. A line

Let $\mathcal{P}$ be a euclideanplan with an orthonormal coordinate system$(O,\ \vec{\imath},\ \vec{\jmath})$

- It is well known that the geometry in the plan seen by Euclid defines that a straight line is the set of aligned points. In mathematics often expressed by the function $f$defined on $\mathbb{R}$ by: $\forall(a,b)\in\mathbb{R}^2$, with $a\neq 0: f(x)=ax+b$ (Jaen LusShabert, 1991).
- But, in the plan seen by Henri Poincaré, one of the founder of non-Euclideanhyperbolicgeometry, whichis none otherthan the upperhalf-plan of the Euclideanplanwith$y\geq 0$, wedefinetwokinds of straight linesincluding:
  - ✓ the verticalhalf-lineslocated on the x -axis. That is to say the wholenoted$D_a$defined by: $D_a=\{(a,y)\in\mathbb{R}^2/a\ :\ \text{constant and } y\geq 0\}$[1] ;
  - ✓ and the semicircles on the upperplancentered on the abscissa axis. That is to say the wholenoted $D_{(a,\ r)}$ defined by: $D_{(a,\ r)}=\{(x,y)\in\mathbb{R}^2/(x-a)^2-y^2=r^2$ where $y\geq 0\}$characteristicfunction$F_{(a,\ r)}(x)=\sqrt{r^2-(x-a)^2}$[2].

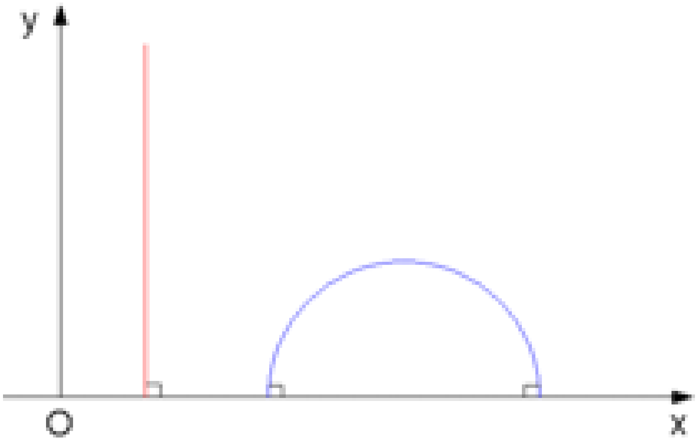

Figure 1 - Two types of lines of Henry Poincaré

2.2. A parabola

A parabola is a conicformed by the intersection of aconewith a planparallel to a generator of the cone. A parabolaisalso the set of points of aplanequidistantfrom a fixed point, the focus, and from a straight line, the directory. The parabola issymmetrical about its axis, straight through the focus and perpendicular to the director. A symmetric parabola with respect to the x-axis and having for vertex the origin has for equation:$y^2 = 2px$in an orthonormal frame, p real positive, being the parameter of the parabola(Bemarisika Parfait, 2004). A veryconvenientparabolicfunction expression in mathematicsisdefined by:$\forall(a, b, c) \in \mathbb{R}^3$ with $\neq 0$: $x \mapsto ax^2 + bx + c$.

III. A TECHNIQUE FOR BUILDING AN ELLIPSE

In this section, we will see or observe a construction technique of an ellipse. Indeed, it is a tool for the study of non-Euclidean geometry according to the Poincaré's model. Moreover, it stands among the most important curves in physics. In astronomy, the orbits of the Earth and other plants around the Sun are ellipses. It is used in engineering for the arches of some bridges and in the design of gears for some machines and even in monumental construction architecture in the shape of an arc.

3.1. Reminder and definition

Definition 1. *An ellipse is a conic in the form of a closed curve, obtained as the intersection of a cone by a plan intersecting a single ply of this cone (Figure 1).*

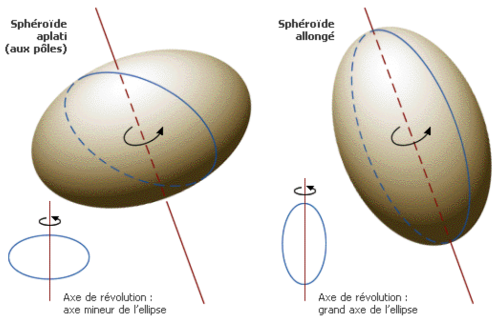

Figure 2 – Examples of ellipses

3.2. Functional approach

Let $\mathcal{P}$ be a Euclidean plan with an orthonormal coordinate system$(O, \vec{\imath}, \vec{\jmath})$.

It is well known that the reduced equation of an ellipse is defined by:

$$\left(\frac{x-a}{q}\right)^2 + \left(\frac{y-b}{r}\right)^2 = 1 \qquad (1)$$

We denote by $\mathcal{E}$ the graphical representation of (1) on $\mathcal{P}$.

Let's start from the expression (1) we have(Armand & André Totohasina, 2019a), (Armand & André Totohasina, 2019b):

$$\left(\frac{x-a}{q}\right)^2+\left(\frac{y-b}{r}\right)^2=1 \Leftrightarrow \left(\frac{r}{q}\right)^2(x-a)^2+(y-a)^2=r^2$$
$$\Leftrightarrow (y-a)^2=r^2-(\alpha)^2(x-a)^2$$
$$\Leftrightarrow y=\pm\sqrt{r^2-(\alpha)^2(x-a)^2}+b, \qquad avec\ \alpha=\frac{r}{q}.$$

If we note by (C_y^+) the graphical representation of the function $y=+\sqrt{r^2-(\alpha)^2(x-a)^2}+b$ and by (C_y^-)thatof$y=-\sqrt{r^2-(\alpha)^2(x-a)^2}+b$, then $\mathcal{E}$= $(C_y^+)\cup(C_y^-)$. The graphicalrepresentation of an ellipse in the plan$\mathcal{P}$is none otherthan the union of twocurvesrepresentative of twofunctions, with one constant, opposite $F^{e+}_{(\alpha,\ a,\ b,\ r)}$ and $F^{e-}_{(\alpha,\ a,\ b,\ r)}$ so defined: $F^{e+}_{(\alpha,\ a,\ b,\ r)}(x)=+\sqrt{r^2-(\alpha)^2(x-a)^2}+b$ and $F^{e-}_{(\alpha,\ a,\ b,\ r)}(x)=-\sqrt{r^2-(\alpha)^2(x-a)^2}+b$ where $(\alpha,\ a,\ b,\ r)\in\mathbb{R}^4$ .Thesetwofunctions are calledcharacteristicfunctions of an ellipse. Note thatbothfunctions$F^{e+}_{(\alpha,\ a,\ b,\ r)}$ and $F^{e-}_{(\alpha,\ a,\ b,\ r)}$each characterizing a half-ellipse, are all defined and well-continuous over the sameboundedinterval$\left[a-\frac{r}{\alpha}\ ;\ a+\frac{r}{\alpha}\right]$(Armand & André Totohasina, 2019), (Armand & André Totohasina, 2019b).

Note 1. *This pedagogicalapproach to the construction of an ellipse would at least have the advantage of avoiding a cognitive break in learnersalreadywellaccustomed to representativecurves of functions and orthogonal symmetry, to a vertical transformation, geometric transformation alreadyacquired at the end of secondaryschool*$C_y^-=S_{(y=b)}(C_y^+)$, *where* $S_{(y=b)}$*is the orthogonal axis symmetry, the equation line* $y=b$ *(Armand & André Totohasina, 2019a), (Armand & André Totohasina, 2019b).*

3.3. Descriptions of the characteristic elements of this ellipse
Recallthat the center of this ellipse is the point *I(a, b).* For the visualization of its axes and itspeaks, itisnecessary to examine the values of realα. Hence the following proposition.

Proposition 1.
- *(i) If*$|\alpha|=1$, *then the union of twocurvesrepresents a circle of center I and radius r;*
- *(ii) If* $|\alpha|<1$, *then the union of twocurvesrepresents a long axis ellipse the abscissa axis of the vertices*$S_1\left(a-\frac{r}{\alpha},\ b\right)S_3\left(a+\frac{r}{\alpha},\ b\right)$, $S_3(a\ ,-r+b)$ *and* $S_4(a\ ,\ r+b)$;
- *(iii) If*$|\alpha|>1$, *then the union of twocurvesrepresents an ellipse of great axis the axis of the ordinates of the vertices*$S_1\left(a-\frac{r}{\alpha},\ b\right)$,$S_3\left(a+\frac{r}{\alpha},\ b\right)$, $S_3(a\ ,-r+b)$ *and* $S_4(a\ ,\ r+b)$.

3.4. Construction of an ellipse whose major axis is located on the y-axis
To understand the construction in question, let'sstartfrom an example. Let us note in passing that for the visualization of elliptic figures using the characteristicfunctions in the plan$\mathcal{P}$, itisadvantageous to combine and to couple the use of the softwares "GeoGebra", "Derive5", "Sine qua non", "Latex with PST + grazing, etc., exploitingtheircomplementarity (Armand & André Totohasina, 2019a), (Armand & André Totohasina, 2019b).

Example 1.*The graphicalrepresentation of the ellipse E1 of the characteristicelements I (0; 2) and vertices*S_1*(-1, 2),* S_2*(1, 2),* S_3*(0, 0),* S_4*(0, 4) isnothing but the union of tworepresentativecurves of twofunctions*$F^{e+}_{(4,\ 0,\ 2,\ 2)}$ *and* $F^{e-}_{(4,\ 0,\ 2,\ 2)}$ *defined by :* $F^{e+}_{(4,\ 0,\ 2,\ 2)}(x)=+\sqrt{2^2-(4x)^2}+2$ *and* $F^{e-}_{(4,\ 0,\ 2,\ 2)}(x)=-\sqrt{2^2-(4x)^2}+2$ *(Figure 7).*

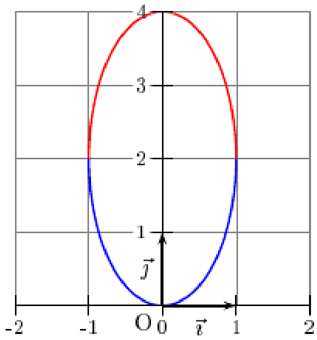

Figure 7 – Ellipse de centre*I(0, 2)*

3.5. Building a circle usingitscharacteristicfunctions

As we have alreadyannounced in paragraph 4 proposition 4.3, the graphicalrepresentation of a circle of center *I(a, b)* and of radius r is no more than the union of twocurvesrepresentative of twofunctions (cf. 12)$F^{c+}_{(1,\ a,\ b,\ r)}$ and $F^{c-}_{(1,\ a,\ b,\ r)}$ defined by : $F^{c+}_{(1,\ a,\ b,\ r)}(x) = +\sqrt{r^2-(x-a)^2}+b$ and $F^{e-}_{(1,\ a,\ b,\ r)}(x) = -\sqrt{r^2-(x-a)^2}+b$.Thesetwofunctions are calledcharacteristicfunctions of a semicircle.

Example 3.*The graphical representation of the circle C of center I(-3, 2) and of radius r = 4 is none other than the union of two curves representative of two functions*$F^{c+}_{(1,-3,\ 2,\ 2)}$ *and* $F^{c-}_{(1,-3,\ 2,\ 2)}$*defined:* $F^{c+}_{(1,-3,\ 2,\ 2)}(x) = +\sqrt{2^2-(x+3)^2}+2$ *and* $F^{e-}_{(1,-3,\ 2,\ 2)}(x) = -\sqrt{2^2-(x+3)^2}+2$*(figure 12)***.**

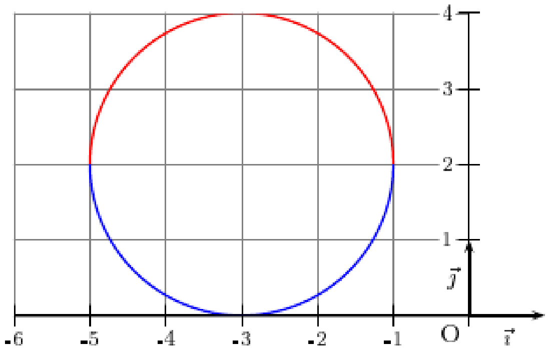

Figure 12 – Circle of center *I (-3; 2)* and radius 3

IV. COMPOSITION OF FUNCTIONS

Let f and $g$two numerical functionsdefinedrespectively on$D_f$and D_g. The compound of$f$by g, denoted gof, isdefined bygof : $x \mapsto gof(x) = g[f(x)]$ for $x \in D_f$ and $f(x) \in D_g$. Subsequently, in mathematics, the composition of functions (or composition of applications) is a processwhichconsists, from at least twofunctions, of constructing a new one. For this, we use the images of the first function as argument for the second (providedthatitmakessense). This iscalled composite function (or compound application).

4.1. Less known composition of some functions

In this section, we speak in particular of the less well-known compound of functions for converting a function to another function via their respective compositions.

4.2. Convertinga circle to a straight line

Let $F^{c+}_{(1,\ a,\ b,\ r)}$a function characterizing an upper semicircle defined on the boundedinterval$[a-r\ ;\ a+r]$ by: $F^{c+}_{(1,\ a,\ b,\ r)}(x) = +\sqrt{r^2-(x-a)^2}+b$.First, to a transformation, thisfunctioncanbewritten$F^{c+}_{(1,\ a,\ b,\ r)}(x) = +r\sqrt{1-(X)^2}+b$with $X = \frac{x-a}{r}$. Recall in passing that if$-1 \leq X \leq 1$, then $a-r \leq x \leq a+r$.For thispurpose, let us nowconsider a function$F_{c\to d}$ defined by: $F_{c\to d}(x) = \sqrt{1-(X)^2}$.

We have

$$F^{c+}_{(1,\ a,\ b,\ r)}oF_{c\to d}(x) = r\sqrt{1-\left(\sqrt{1-(X)^2}\right)^2}+b$$
$$= rX+b$$
$$= x+b-a$$
$$= x+\lambda \text{with } \lambda = b-a.$$

Morality 2. *The function*$F_{c\to d}$definediscalled the conversion function to transform the functions$F^{c+}_{(1,\ a,\ b,\ r)}$ *and* $F^{c-}_{(1,\ a,\ b,\ r)}$characterizing a circle in affine function. In otherwords, wewillthinkthatthisapproachcanovercome the problem of trajectory conversion of a homogeneous body undergoes a circular motion in rectilineartrajectorybecauseitstrajectoryisinitiallygenerated by a circle of radius r and center *I(a, b)*, thenitsrectilineartransform has for linetrajectorysuchthat$x \mapsto x+\lambda$,*with*$\lambda = b-a$.

4.3. Converting a line to a circle

This time, let'sstartfrom the functiong_d defined by: $g_d(x) = \varepsilon x + \delta$.Subsequently, the conversion of a line to a circle consists of transforming the function$g_d$to the function$F^{c+}_{(1,\ a,\ b,\ r)}$or $F^{c-}_{(1,\ a,\ b,\ r)}$. It's about building and looking for a transformation $F_{d\to c}$ for example)whichcanmakethisfunction affine to a function$F^{c+}_{(1,\ a,\ b,\ r)}$or$F^{c-}_{(1,\ a,\ b,\ r)}$. For thispurpose, let us nowconsider a function$F_{d\to c}$ defined by: $F_{d\to c}(x) = \sqrt{r^2 + (x/\varepsilon)^2} + b$.This functionisused to transform the function g_d in a circle of radius r and center *I(0; b)*suchthat: $g_d(x) = \varepsilon x + \delta$ and $F_{d\to c}(x) = \sqrt{r^2 + (x/\ \varepsilon)^2} + b$ we have:

$$F_{c\to d} o g_d(x) = \sqrt{r^2 - \left(\frac{\varepsilon\left(x + \frac{\delta}{\varepsilon}\right)}{\varepsilon}\right)^2} + b$$

$$= \sqrt{r^2 - \left(x + \frac{\delta}{\varepsilon}\right)^2} + b$$

$$= \sqrt{r^2 - (x - a)^2} + b \text{with} a = -\frac{\delta}{\varepsilon}$$

Morality 3. *The function*$F_{d\to c}$thusdefinediscalled the conversion functionwhichmakesit possible to transform an affine function to the function $F^{c+}_{(1,\ a,\ b,\ r)}$ *or* $F^{c-}_{(1,\ a,\ b,\ r)}$. That is to say, thistheorycouldbeconverted the trajectory of ahomogeneous body undergoes a rectilinear motion in a circulartrajectory.

4.4. Converting a line to a parabola

This conversion is not a problem. Indeed the compound of agiven line of the type$x \mapsto \varepsilon x + \delta$by a parabola of the type: $x \mapsto ax^2 + bx + c$and vice versa gives a corresponding parabola.

4.5. Conversion of a dish to a straight line

Let f a parabolicfunction set on $\mathbb{R}$ by: $\forall x \in \mathbb{R}$, $f(x) = ax^2 + bx + c$ where $a \neq 0$. The conversion of thisfunctioninto a line via the composition of a givenfunctioncomes down to finding a function $f_{p\to d}$, for example, as$f o f_{p\to d}$give a line. For thispurpose, wecanwrite$f_{p\to d}(x) = \sqrt{x} - \frac{b}{2a}$.

Indeed, on has:

$$f(x) = ax^2 + bx + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \gamma.$$

So,

$$f o f_{p\to d}(x) = a\left(\sqrt{x} - \frac{b}{2a} + \frac{b}{2a}\right)^2 + \gamma$$

$$= a|x| + \gamma.$$

We find that the last expression is a straight line.

4.6. Converting a parabola to a circle

Let f a parabolic function set on $\mathbb{R}$ by: $\forall x \in \mathbb{R}$, $f(x) = ax^2 + bx + c$ where $a \neq 0$. The conversion of thisfunctioninto a line via the composition of a givenfunctioncomes down to finding a function $f_{p\to c}$, for example, as$f o f_{p\to c}$give a line. For thispurpose, wecanwrite$f_{p\to c}(x) = \sqrt{r^2 - \frac{x}{a} - \gamma} + \chi$.

Indeed, on has:

$$f(x) = ax^2 + bx + c$$

$$= a\left(x + \frac{b}{2a}\right)^2 + \gamma.$$

So, on gets:

$$f_{p\to d} o f(x) = \sqrt{r^2 - \frac{a\left(x + \frac{b}{2a}\right)^2 + \gamma}{a} - \gamma + \chi}$$

$$= \sqrt{r^2 - a\left(x + \frac{b}{2a}\right)^2} + \chi$$

$$= \sqrt{r^2 - a(x - \omega)^2} + \chi.$$

The last relationshipgives us the semicircleequationgenerated by the center circle$W(\omega,\ \chi)$and radius$r$.

4.7. Converting a circle to a parabola

Let $F^{c+}_{(1,\ a,\ b,\ r)}$a function characterizing an upper semicircle defined on the boundedinterval$[a-r\ ;\ a+r]$ by: $F^{c+}_{(1,\ a,\ b,\ r)}(x) = +\sqrt{r^2-(x-a)^2}+b$ .This time, wewill look for the function $F_{c\to p}$ for example, as $F_{c\to p}oF^{c+}_{(1,\ a,\ b,\ r)}(x) = Ax^2+Bx+C$ with $(A,B,C)\in\mathbb{R}^3$ such as $A\neq 0$. Wecanindeedtake for example$F_{c\to p}(x) = \psi(x-b)^2-\psi r+\rho$.

And we have:

$$F_{c\to p}of(x) = \psi\left(\sqrt{r^2-(x-a)^2}+b-b\right)^2-\psi r+\rho$$
$$= -\psi x^2-2a\psi x-a^2+\rho$$

$= Ax^2+Bx+C$with$(A,B,C)\in\mathbb{R}^3$suchas$A\neq 0$.

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V. CONCLUSION AND PERSPECTIVES

In a nutshell, it is important to introduce this approach in order to solve situations of the various problems to be determined, to search, to calculate as well as to justify what we need. In this work, we have also tried to make some reflections through the stakes of the composition of numerical functions of on real variable. We believe that this approach could give the possible application of said compositions from general to particular for each area where they are needed. It would facilitate the classification of functions that can be used for other functions. We have also tried to show in this modest work some examples of situation arguing the necessity of mathematics in the modeling of the conversion of some function via the composition. We have also shown some examples of situations giving reasons or evidence that justify or support a less well-known composition point of view of the few functions through the possible resolution of the few mathematical problems. In the future work, we will think to give the contribution of this approach through the study of rectilinear movement's uniform, uniformly varied and circular including the transformations of the trajectories of these movements

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