International Journal of Multidisciplinary and Current Educational Research (IJMCER)
ISSN: 2581-7027 ||Volume|| 1 ||Issue|| 4 ||Pages|| 01-07 ||2019||

Notes on Non-orthogonal Axial Symmetry: Complex and Analytical Expression

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Abstract:*The symmetry is a property of certain plane figures which remain unchanged during transformations made with respect to a point, an axis or a plane. It also represents an involutional geometric transformation that maintains parallelism. In particular, the present work explains a very explicit geometric method giving the analytical and complex expression of a non-orthogonal axial symmetry. An orthogonalsymmetry is its particular case.*

KEY WORDS*: non-orthogonal axial symmetry, analytical and complex expression.*

I- INTRODUCTION

Symmetry is the property of a system when two parts are similar,usually with respect to a point or an axis or a plane. The best-known example is the symmetry in geometry. In general, a system is symmetrical when one can switch its elements leaving its shape unchanged [18], [7]. The concept of automorphism makes it possible to specify this definition. A butterfly, for example (see Figure 1), is symmetrical because one can swap all the points of the left half of one's body with all the points of the line half without its appearance being modified.

Figure 1 – Photo of butterfly

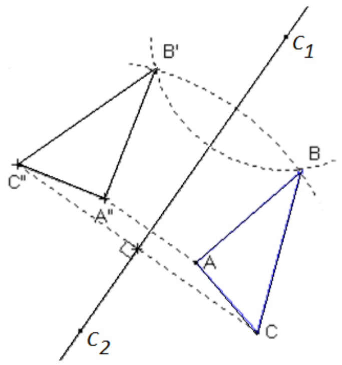
We can exchange the two halves without changing the shape of the set. Symmetrical figures make visible the equality of forms because the permutable parts always have the same form. We could make a definition of the concept: a figure is symmetrical when it repeats the same form on a regular basis [8], [10], [1], [19]. On this subject, we contribute in this paper the very explicit geometric method of the analytical and complex expression of the non-orthogonal axial symmetries generating from that of orthogonal. Indeed, learning about these expressions is always a subject sincerely to be exploited by way of improvement. In fact, in geometry, symmetry also constitutes property of certain plane figures which remain unchanged during transformations made with respect to a point, an axis or a plane. It also represents an involutional geometric transformation that maintains parallelism. There are several kinds of symmetries in the plane or in space. Note that the term symmetry also has another meaning in mathematics. Common symmetries include reflection and central symmetry. This term designates either a translation, an orthogonal automorphism, or the compound of the two [16], [12], [17], [6]. In terms of learning the geometries in terminal S, this fact still remains in a way not expressly formulated [7], [4], [5], [9], [13], [14], [15], [3]. In what follows, our work is divided into five sections. Section 7 gives some symmetries existing in the literature. Section 3 presents the analytical and complex expression of non-orthogonal axial symmetry. Section 4 introduces the opposite problem of Section 3. Section 5 concludes.

II- DIFFERENT KINDS OF SYMMETRIES

In this section, we will see the different kinds of geometric symmetries that exist.

2.1. Axial or orthogonal symmetry with respect to a straight line

They are also called axis reflections (C_1C_2). The reflection of axis (C_1C_2) is the transformation of the plane which leaves all the points of (C_1C_2) invariants and which, at any point B not situated on (C_1C_2), associates the point B'such that (C_1C_2) is the mediator of$[BB']$. Since there are two equivalent definitions of the mediator, we thus know two equivalent constructions of the pointB' (Cf. figure 2).



2.2. Central symmetry (relative to a point)

The symmetry of center O is the transformation which, at any point C, associates the point C' such that O be the middle of$[CC']$ (cf. figure 2).

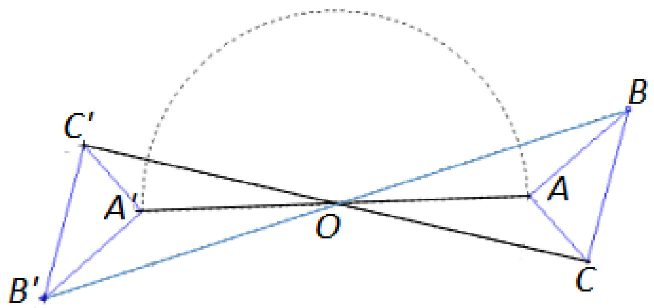


Figure 3 – Centrale symmetry

Let us observe that the only invariant point of this symmetry is the point O. A symmetry of center O is also a rotation of flat angle and a homothety of center O and ratio1.

2.3. Group of central symmetries-translations

As the compound of two symmetries of centers O and O' is a vector translation$2\overrightarrow{OO'}$.

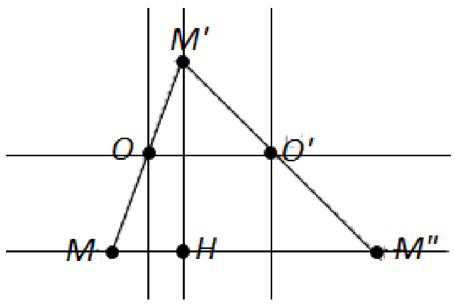


Figure 4 – Symmetry group

The theoremof the middle makesit possible to noticethat$\overrightarrow{MM"} = 2\overrightarrow{OO'}$. This propertymakesit possible to define a first group of transformations of the plane: that of the central symmetries-translations. Indeed, by composingtwo central symmetries or translations, weobtain a central symmetry or a translation. And, to get the identical application, simply compose avector translation$\vec{U}$by the vector translation$\vec{U}$, or to compose a central symmetry by itself.

2.4. Non-orthogonal axial symmetry

We find the same definition and the same properties as for the central symmetry in the plane, except that a central symmetry does not maintain the orientation in space. Thus, the guy raises his right hand and his image lifts his left hand (Cf. figure 5).

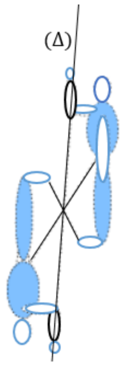


Figure 5 – Central symmetry

About this symmetry, less subject deals with the generality of its analytical and complex expression. We will see them in the next section.

III- ANALYTICAL EXPRESSIONS AND COMPLEX AXIAL SYMMETRY

Note that our goal is to give the general, analytical and complex expression of axial axis symmetry Δdenoted$S_\Delta$generating the general, analytical and complex expression of non-orthogonal axis symmetry Δ (for example) which still a subject that should be exploited. The plane $\mathcal{P}$ is provided with an orthonormal reference$(O,\ \vec{\imath},\ \vec{\jmath})$. In this plane, we consider an axial non-orthogonal symmetry(S_Δ) and an orthogonal symmetry $(S_{\Delta'})$ respectively (Δ) and (Δ') such as (Δ) is the image of (Δ') by a rotation r of angle$\theta = \frac{\pi}{2} - \alpha$and having respective director vectors$\vec{U}(-b, a)$and $\vec{U}'(-b', a')$. Consider the two points M and $M'$of P such that$S_{\Delta,\alpha}(M) = M'$ and $S_{\Delta',\frac{\pi}{2}}(M) = M'$(cf. figure 6).

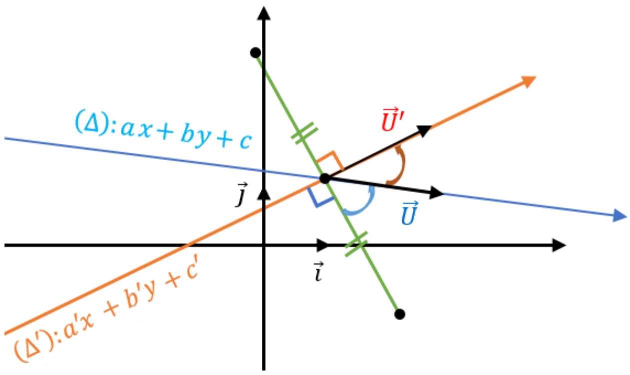


Figure 6 – Axial symmetry

Let JandJ' be two points such as$\overrightarrow{IJ} = \vec{U}$ and $\overrightarrow{IJ'} = \vec{U}'$such as$mes\left(\overrightarrow{IJ}, \overrightarrow{IJ'}\right) = \theta$(Cf. Figure 6). These confirm that

$$\begin{cases} z_J - z_I = -b + ia \\ z_{J'} - z_I = -b' + ia' \end{cases}.$$

Now, the image of the line$\Delta$by the angle rotation$\theta$and center I is the lineΔ'. Which means that:

$$z_{J'} - z_I = (z_j - z_I)e^{i\theta} \quad (1)$$

For $\theta = \frac{\pi}{2} - \alpha$where $\alpha = \left(\overrightarrow{MM'}, \vec{U}\right)$, thisrelationshipallows us to have the relationship (2) defined by:

$$\begin{cases} a' = -bcos\alpha + asin\alpha \\ b' = bsin\alpha + acos\alpha \end{cases} \quad (2)$$

Note 1. *Wenoticedthat if the line (Δ) is (Δ′) whichis orthogonal to the segment*$[MM']$ *then* $\alpha = \frac{\pi}{2}$*i.e.* $b' = b$*and* $a' = a$.

As $I$is the middle of segment $[MM']$, itfollowsthat$I\left(\frac{x'+x}{2}, \frac{y'+y}{2}\right)$, $I \in (\Delta)$ and $I \in (\Delta')$.

Point$I \in (\Delta)$is equivalent to

$$a(x + x') + b(y + y') + 2c = 0 \quad (3)$$

Furthermore, $I \in (\Delta')$ then

$$a'(x + x') + b'(y + y') + 2c' = 0 \quad (4)$$

As well,

$$\begin{cases} \overrightarrow{MM'} \perp \Delta \Leftrightarrow \overrightarrow{MM'} \perp \vec{U} \\ \qquad\qquad \Leftrightarrow \overrightarrow{MM'}.\vec{U} = 0 \end{cases} \quad (5)$$

which is equivalent to:

$$\begin{pmatrix} x' - x \\ y' - y \end{pmatrix}\begin{pmatrix} -b' \\ a' \end{pmatrix} = 0 \quad (6)$$

The two relations (3) and (6) lead us to have the expression (7) called the general analytical expression of anon-orthogonal axis symmetry (Δ) of the angle α (Cf. Figure 6).

$$\begin{cases} x' = \frac{1}{aa'+bb'}[-(aa' - bb')x - 2a'by - 2a'c] \\ y' = \frac{1}{aa'+bb'}[-2ab'x + (aa' - bb')y - 2b'c] \end{cases} \quad (7)$$

By playing on the two equations (4) and (7) we have the expression (8) by:

$$c' = \frac{1}{aa'+bb'}[(a'b - b'a)(b'x - a'y) - c(a'^2 + b'^2)] \quad (8)$$

Moreover, the expression (7) leads us to have the expression (9) called general complex expression of a non-orthogonal axis symmetry (Δ) of the angle $\alpha = \left(\overrightarrow{MM'}, \vec{U}\right)$defined by:

$$z' = \frac{1}{aa'+bb'}[(aa' - bb' + 2iab)\bar{z} + 2a'c + 2ib'c] \quad (9)$$

Consequently, without however foreseeing what future research will give, we will state in the form of a theorem.

Theorem 1.*Let* $(\Delta) : ay + bx + c = 0$ *and* $(\Delta') : a'y + b'x + c' = 0$ two respective directorvectorlines $\vec{U}(-b, a)$ *and* $\vec{U'}(-b', a')$ *with* $\left(\vec{U}, \vec{U'}\right) = \theta$, M *and* M' are two distinct points of $\mathcal{P}$suchthat$S_{\Delta,\alpha}(M) = M'$ *and* $S_{\Delta'}(M') = M$. *Furthermore,* S_Δ *and* $S_{\Delta'}$ *are non-orthogonal axis symmetry* Δ*and orthogonal axis symmetry*Δ'*respectivelyand* $\alpha = \left(\overrightarrow{MM'}, \vec{U}\right)$*(cf. figure 6). If*$(\Delta') : a'y + b'x + c' = 0$*is the orthogonal symmetry image of*(Δ)*by a rotation r of center I of angle*θ*, then the generalanalytical expression of this axial symmetry isdefined by:*

$$\begin{cases} x' = \frac{1}{aa'+bb'}[-(aa' - bb')x - 2a'by - 2a'c] \\ y' = \frac{1}{aa'+bb'}[-2ab'x + (aa' - bb')y - 2b'c] \end{cases} \quad (10)$$

and it has a general complex expression defined by:

$$z' = \frac{1}{aa' + bb'}[(aa' - bb' + 2ia'b)\bar{z} + 2a'c + 2ib'c] \quad (11)$$

with

$$\begin{cases} a' = -b\cos\alpha + a\sin\alpha \\ b' = b\sin\alpha + a\cos\alpha \end{cases}$$

and

$$c' = \frac{1}{aa' + bb'}[(a'b - b'a)(b'x - a'y) - c(a'^2 + b'^2)] \quad (12)$$

IV- INVERSE PROBLEME

Wewillnowsee the inverse problem of section III. That is, given the complex expression of axis symmetry(Δ) : $ay + bx + c = 0$,how to calculate the coefficients *a*, *b*, *c*of the analytical expression of (Δ) : $ay + bx + c = 0$. The followingcorollaryallowsus to express it. Note thatthiscorollaryisderivedfrom Theorem1 above.

Corollary 1. *Expression (11) canbewritten in form (13) defined by:*

$$z' = \mathcal{A}\bar{z} + \mathcal{B} \quad (13)$$

And

$$\begin{cases} \mathcal{Re}(\mathcal{A}) = -\frac{aa' - bb'}{aa' + bb'} \\ \mathcal{Im}(\mathcal{A}) = -\frac{2a'b}{aa' + bb'} \\ \mathcal{Re}(\mathcal{B}) = -\frac{2a'c}{aa' + bb'} \\ \mathcal{Im}(\mathcal{B}) = -\frac{2b'c}{aa' + bb'} \end{cases} \quad (14)$$

With expression (14), wecansearch for the equation of (Δ) defined by (Δ): $ay + bx + c = 0$ knowing the complex expression of $S_\Delta$defined on(13).

Corollary 2. *As we have alreadynoticedthat if the line(Δ) is (Δ')which is orthogonal to the segment$[MM']$ then $\alpha = \frac{\pi}{2}$i.e. $b' = b$and $a' = a$ (C.f Figure 7).*

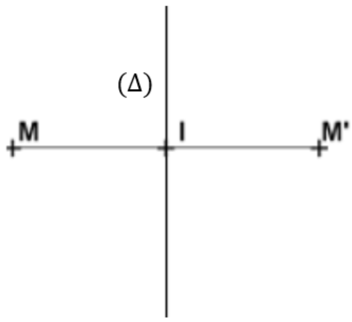


Figure 7 – Orthogonal axis symmetry (Δ)

In which case, the expression (10) becomes (15) defined by:

$$\begin{cases} x' = \frac{1}{a^2 + b^2}[-(a^2 - b^2)x - 2aby - 2ac] \\ y' = \frac{1}{a^2 + b^2}[-2abx + (a^2 - b^2)y - 2bc] \end{cases} \quad (15)$$

On thissubject, we have the analytical expression of orthogonal axis symmetry S_Δ defined on (15) above.Moreover, the expression(15) leads us to have the expression (16) calledcomplex expression ofS_Δ by:

$$z^{'} = \frac{1}{a^2+b^2}\left[(a^2 - b^2 + 2iab)\bar{z} + 2ac + 2ibc\right] \quad (16)$$

Fromwhere the corollary 3 makesit possible to look for the coefficients a, b, $c$structuring the equation ofΔ defined by $(\Delta) : ay + bx + c = 0$knowing the complex expression of S_Δ defined on (16).

$$z^{'} = \mathcal{A}\bar{z} + \mathcal{B} \quad (17)$$

$$\begin{cases} \mathcal{Re}(\mathcal{A}) = -\frac{a^2-b^2}{a^2+b^2} \\ \mathcal{Im}(\mathcal{A}) = -\frac{2a^{'}b}{a^2+b^2} \\ \mathcal{Re}(\mathcal{B}) = -\frac{2ac}{a^2+b^2} \\ \mathcal{Im}(\mathcal{B}) = -\frac{2bc}{a^2+b^2} \end{cases} \quad (18)$$

Wenow have a method to calculate the coefficients a, b, $c$structuring the equation of(Δ) defined by $(\Delta) : ay + bx + c = 0$knowing the complex expression ofS_Δ defined on (16).

V- CONCLUSION AND PERSPECTIVES

This work allows us to know the concept of some forms defined in mathematics from that of isomorphism. Two isomorphic systems have the same shape. A system, a mathematical structure, a model, a universe, or a world, in the mathematical sense, is determined with several sets, namely a given set of elements of the system, its points or its elementary constituents, the set of fundamental predicates, basic properties of the elements and relationships between them, the set of operators, or functions, which further determine the structure of the system. Often by misuse of language, one identifies a structure by a given set of its elements. It is also important to note that this work allowed us to solve problems of general, analytical and complex expression, axial symmetry generating the general, analytical and complex expression of orthogonal symmetry. We will try to emit some reflections of this approach on the symmetries of molecules and crystals based mainly on a molecule or crystal is defined in quantum mechanics by the wave function of all its constituents, nuclei and electrons. But for many uses, one can model the structure simply by the positions of the centers of atoms or ions. The notions of symmetries of the molecule or of the crystal can indeed be defined as the isometries of the space which are also automorphisms for the monadic predicates of structure. Also, we will consider in its application on the extension of the notions of the studies on the periodic trajectories (the oscillations, the vibrations, the movements of the satellites) which are spatio-temporal symmetrical structures for some translations in the time

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